Young, Thomas. “Bridge”. En: *Supplement to the fourth, fifth and sixth editions of the Encyclopaedia Britannica.* Edinburgh: Archibald Constable, 1824. Vol. 2, pp. 497-520, plates 42, 43, 44.

and brown, according to the substances with which the oxide is combined. We ascribe the yellow colour of the London bricks to the ashes of the coals, which, by uniting with the peroxides of iron, form a kind of yellow ochre. (J.)

Brick || Bridge.

BRIDGE.

The mathematical theory of the structure of bridges has been a favourite subject with mechanical philosophers; it gives scope to some of the most refined and elegant applications of science to practical utility; and at the same time that its progressive improvement exhibits an example of the very slow steps by which speculation has sometimes followed execution, it enables us to look forwards with perfect confidence to that more desirable state of human knowledge, in which the calculations of the mathematician are authorised to direct the operations of the artificer with security, instead of watching with servility the progress of his labours.

Of the origin of the art of building bridges a sketch has been given in the body of the *Encyclopædia;* the subject has been rediscussed within the last twenty years by some of the most learned antiquaries, and of the most elegant scholars of the age; but additions still more important have been made to the scientific and practical principles on which that art depends; and the principal information, that is demanded on the present occasion, will be comprehended under the two heads of physico-mathematical principles, subservient to the theory of this department of architecture, and a historical account of the works either actually executed or projected, which appear to be the most deserving of notice. The first head will contain three sections, relating respectively (1) to the resistance of the materials employed, (2) to the equilibrium of arches, and (3) to the effects of friction; the second will comprehend (4) some details of earlier history and literature, (5) an account of the discussions which have taken place respecting the improvement of the port of London, and (6) a description of some of the most remarkable bridges which have been erected in modern times.

Section I.—*Of the Resistance of Materials.*

The nature of the forces on which the utility of the substances employed in architecture and carpentry depends, has been pretty fully investigated in the article Strength of the *Encyclopædia;* and the theory has been carried somewhat further, in the investigations of a late writer concerning Cohesion and Passive Strength of materials. Much, however, still remains to be done; and we shall find many cases, in which the principles of these calculations admit of a more immediate and accurate application to practice than has hitherto been supposed. It will first be necessary to advert to the foundation of the theory in its simplest form, as depending on the attractive and repulsive powers, which balance each other, in all natural substances remaining in a permanent state of cohesion, whether as liquids, or as more or less perfect solids.

A. *In all homogeneous solid bodies, the resistances to extension and compression must be initially equal, and proportional to the change of dimensions.*

The equilibrium of the particles of any body remaining at rest, depends on the equality of opposite forces, varying according to certain laws; and that these laws are continued without any abrupt change, when any minute alteration takes place in the distance, is demonstrated by their continuing little altered by any variation of dimensions, in consequence of an increase or diminution of temperature, and might indeed be at once inferred as highly probable, from the general principle of continuity observed in the laws of nature. We may, therefore, always assume a change of dimensions so small, that, as in all other differential calculations, the elements of the curves, of which the ordinates express the forces, as functions of, or as depending on, the distances as abscisses, may be considered as not sensibly differing from right lines, crossing each other, if the curves be drawn on the same side of the absciss, in a point corresponding to the point of rest, or to the distance affording an equilibrium; so that the elementary finite differences of the respective pairs of ordinates, which must form, with the portions of the two curves, rectilinear triangles, always similar to each other, will always vary as the lengths of the elements of the curves, or as the elements of the absciss, beginning at the point of rest; and it is obvious that these differences will represent the actual magnitude of the resistances exhibited by the substance to extension or compression. (Plate XLII. fig. 1.)

It was on the same principle that Bernoulli long ago observed, that the minute oscillations of any system of bodies, whatever the laws of the forces governing them might be, must ultimately be isochronous, notwithstanding any imaginable variation of their comparative extent, the forces tending to bring them back to the quiescent position being always proportional to the displacements; and so far as the doctrine has been investigated by experiments, its general truth has been amply confirmed; the slight deviations from the exact proportion, which have been discovered in some substances, being far too unimportant to constitute an exception, and merely tending to show that these substances cannot have been perfectly homogeneous, in the sense here attributed to the word. When the compression or extension is considerable, there may indeed be a sensible deviation, especially in fibrous or stratified substances; but this irregularity by no means affects the admissibility of any of the conclusions which will be derived from this proposition.

B. *The strength of a block or beam must be reduced to one half, before its cohesive and repulsive forces can both be called into action.*

We must suppose the transverse sections of the body to remain plane and perpendicular to the axis, whatever the point may be to which the force is applied, a supposition which will be correctly true, if the pressure be made by the intervention of a firm plate attached to each end, and which is perfectly admissible in every other case. Now, if the terminal plates remain parallel, it is obvious that the compression or extension must be uniformly distributed throughout the substance, which must happen when the original force is applied in the middle of the block; the centre of pressure or resistance, collected by the plate, acting like a lever, being then coincident with the axis. But when the plates are inclined, the resistance depending on the compression or extension will be various in different parts, and will always be proportional to the distance from the neutral point, where the compression ends and the extension begins, if the depth of the substance is sufficient to extend to this point; consequently the forces may always be represented, like the pressure of a fluid, at different depths, by the ordinates of a triangle; and their result may be considered as concentrated in the centre of gravity of the triangle, or of such of its portions as are contained within the depth of the substance; and when both extension and compression are concerned, the smaller force may be considered as a negative pressure, to be subtracted from the greater, as is usual when any other compound forces are supposed to act on a lever of any kind. Now, when the neutral point is situated in one of the surfaces of the block, the sum of all the forces is represented by the area of the triangle, as it is by that of the parallelogram when the plates remain parallel, and these areas being in either case equivalent to the same external force, it is obvious that the perpendicular of the triangle must be equal to twice the height of the parallelogram, indicating that the compression or extension of the surface in the one case is twice as great as the equable compression or extension in the other; and since there is always a certain degree of compression or extension which must be precisely sufficient to crush or tear that part of the substance which is immediately exposed to it, and since the whole substance must in general give way when any of its parts fail, it follows that the strength is only half as great in the former case as in the latter. And the centre of gravity of every triangle being at the distance of one-third of its height from the base, the external force must be applied, in order to produce such a compression or extension, at the distance of one sixth of the depth from the axis; and when its distance is greater than this, both the repulsive and cohesive forces of the substance must be called into action, and the strength must be still further impaired. (Plate XLII. fig. 2.)

C. *The compression or extension of the axis of a block or beam is always proportional to the force, reduced to the direction of the axis, at whatever distance it may be applied.*

We may suppose one of the inflexible plates, attached to the extremities of the block, to be continued to the given distance, and to act as a lever held in equilibrium by three forces, that is, by the cohesive and repulsive resistances of the block, and the external force; and it is obvious that, as in all other levers, the external force will always be equal to the difference of the other two forces depending on the compression and extension, or to the mean compression or extension of the whole, which must also be the immediate compression or extension of the middle, since the figure representing the forces is rectilinear. And the effect will be the same, whatever may be the intermediate substances by which the force is impressed on the block, whether continued in a straight line or otherwise. When the force is oblique, the portion perpendicular to the axis will be resisted by the lateral adhesion of the different strata of the block, the compression or extension being only determined by the portion parallel to the axis; and when it is transverse, the length of the axis will remain unaltered. But the line of direction of the original force must always be continued till it meets the transverse section at any point of the length, in order to determine the nature of the strain at that point.

D. *The distance of the neutral point from the axis of a block or beam is to the depth, as the depth to twelve times the distance of the force, measured in the transverse section.*

Calling the depth a, and the distance of the neutral point from the axis z, the resistances may be expressed by the squares of $\frac{1}{2}a+z$ and $\frac{1}{2}a-z$, which are the sides of the similar triangles denoting the compression and extension (Prop. B.); consequently, the difference of these squares, $2az$, will represent the external force (Prop. C.). But the distance of the centres of gravity of the two triangles must always be $\frac{2}{3}a$; and, by the property of the lever, making the centre of action of the greater resistance the fulcrum, as the external force is to the smaller resistance, so is this distance to the distance of the force from the centre of action of the greater resistance; or $2az : (\frac{1}{2}a+z)^2 = \frac{2}{3}a : \left(\frac{aa}{12z}-\frac{a}{3}+\frac{z}{3}\right)$; and adding to this the distance of the centre of action from the axis, which must be $\frac{1}{2}a-\frac{1}{3}(\frac{1}{2}a+z) = \frac{1}{3}a-\frac{1}{3}z$, we have $\frac{aa}{12z}$ for the distance of the force from the axis; whence, calling this distance y, $z=\frac{aa}{12y}$.

E. *The power of a given force to crush a block is increased, by its removal from the axis, supposing its direction unaltered, in the same proportion as the depth of the block is increased by the addition of six times the distance of the point of application of the force, measured in the transverse section.*

Since the compression or extension of the axis is invariable, whatever the distance of the force may be, that of the nearest surface must be as much greater, by the properties of similar triangles, as the half depth, increased by the distance of the neutral point, is greater than that distance itself, that is, in the ratio of $a + 6y$ to a, since z is to a as a to $12y$; (Prop. D.) and to $\frac{1}{2}a$ as a to $6y$: and the strength is reduced in the same proportion, as the partial compression or extension, by the operation of a given force, is increased. (Plate XLII. fig. 3.)

F. *The curvature of the neutral line of a beam at any point, produced by a given force, is proportional to the distance of the line of direction of the force from the given point of the axis, whatever that direction may be.*

Since the distance z of the neutral point from the axis is inversely as y, the distance of the force, and the radius of curvature, or the distance of the intersection of the planes of the terminal plates from the neutral point, must be to the distance z as the whole length of the axis is to the alteration of that length produced by the compression or extension, it follows that the radius of curvature must be inversely as the distance y, and inversely also as the compression, and the curvature itself must be conjointly as the force and as the distance of its application. If the direction of the force be changed, and the perpendicular falling from the given point of the axis on the line of the force be now called y, the distance of the force from the axis measured in the transverse section will be increased by the obliquity exactly in the same ratio as its efficacy is diminished, and the curvature of the neutral line will remain unaltered; although the place of that line will be a little varied, until at last it coincides with the axis, when the force becomes completely transverse: and the radius of curvature of the axis will always be to that of the neutral line as the acquired to the original length of the axis. (Plate XLII. fig. 4.)

G. *The radius of curvature of the neutral line is to the distance of the neutral point as the original length of the axis to the alteration of that length; or as a certain given quantity to the external force: and this quantity has been termed the Modulus of elasticity.*

Or $r : z = M : f$, and $r = \frac{Mz}{f} = \frac{Maa}{12fy}$, as is obvious from the preceding demonstration; y being the distance of the line of the force from the given point, whatever its direction may be.

H. *The flexibility, referred to the direction of the force, is expressed by unity, increased by twelve times the square of the distance, divided by that of the depth.*

Making the alteration of the axis unity, the corresponding change at the distance y will be to 1 as $z + y$ to z, or as $1 + \frac{y}{z}$ to 1, and will consequently be equal to $1 + \frac{12yy}{aa}$ (Prop. D.)

When the direction of the force becomes oblique, the actual compression of the axis is diminished, but its effect referred to that direction remains unaltered.

I. *The total compression of a narrow block, pressed in the direction of one of its diagonals, is twice as great as if the same force were applied in the direction of the axis.*

This proposition affords a simple illustration of the application of the preceding one. Calling the length of any portion of the axis x, beginning from the middle, and neglecting the obliquity, the distance of the force may be called $y = nx$, and the compression in the line of the force being everywhere as $1 + \frac{12yy}{aa}$, its fluxion will be $\dot{x} + \dot{x}\frac{12nnxx}{aa}$, and the fluent $x + \frac{4n^2x^3}{aa}$, which, when $y = \frac{1}{2}a$, becomes $x + x$, which is twice as great as if y were always $= 0$. But if the breadth of the block were considerable, so that it approached to a cube, the compression would vary according to a different law, each section parallel to the diagonal affording an equal resistance, and the exact solution of the problem would require an infinite series for expressing the value of $\int n^{\frac{1}{x}} \dot{x}$.

K. *If a solid bar have its axis curved a little into a circular form, and an external force be then applied in the direction of the chord, while the extremities retain their angular position, the greatest compression or extension of the substance will ultimately be to the mean compression or extension which takes place in the direction of the chord, as $1 + \frac{4h}{a}$ to $1 + \frac{16hh}{15aa}$; a being the depth of the bar, and h the actual versed sine, or the height of the arch.*

We must here separate the actions of the forces retaining the ends of the bar into two parts, the one simply urging the bar in the direction of the chord, and the other, which is of a more complicated nature, keeping the angular direction unaltered; and we must first calculate the variation of the angular situation of the ends, in consequence of the bending of the bar by the first portion, and then the strain required to obviate that change, by means of a force acting in the direction of the middle of the bar, while the ends are supposed to be fixed. If each half of the bar were rectilinear, these two strains would obviously be equal, and would neutralise each other in the middle of the halves, which might be considered as the meeting of the ends of two shorter pieces, acting transversely or obliquely on each other; without any strain; the curvature produced by the whole strain being elsewhere as the distance from the line joining these points. But, since the bar is supposed to be curved, it becomes necessary to determine the place of these neutral points, by calculating the change of its angular position throughout its extent.

Considering, first, the middle of the bar as fixed, and calling the angular extent of the variable arc x, beginning from the middle, and the radius r, the ordinate y, or the distance of the arc from the chord, will be $r\varsigma x - b$, b being the cosine of the whole arc; and the fluxion of the change of the angular situation, being as the strain and the fluxion of the arc conjointly, will be expressed by $pr\varsigma x\dot{x} - pb\dot{x}$, of which the fluent is $pr\int x - pbx$. In the second place, the curvature derived from the force acting between the two halves, when the ends are considered as fixed points, will be as $r - r\varsigma x$, and the fluent of the change of angular situation may be called $qrx - qr\int x$; and at the end, when x becomes equal to c, the whole extent of the arc, these two deviations must destroy each other, since the positions of the middle and of the ends remain unaltered; consequently $pr\int c - pbc = qrc - qr\int c$, whence $\frac{p}{q} = \frac{rc - r\int c}{r\int c - bc}$; and the exact proportion of p to q may be found, by means of a table of sines. But when the arc is small, $\int c$ being equal to $c - \frac{1}{6}c^3 + \frac{1}{120}c^5 \ldots$, $rc - r\int c$ is $\frac{1}{6}rc^3$, and $r\int c - bc = (r-b)\,c - \frac{1}{6}rc^3$; now $r-b$, the versed sine of the arc, becomes ultimately $\frac{1}{2}rc^2$, and $(r-b)\,c = \frac{1}{2}rc^3$; therefore $p : q = \frac{1}{6} : \frac{1}{3} = \frac{1}{2} : 1$; that is, the strain at the middle, expressed by p, must be half as great as the strain at the ends, expressed by q: consequently, when the force is considered as single, the distance of the line of its direction from the summit must ultimately be one-third of the versed sine or height.

Now if we call any portion of the chord x, we have for the corresponding value of y, the distance from the line of direction of this force, $\sqrt{(r^2 - x^2)} - d$, and for the fluxion of the compression or extension in the direction of the chord, $\mathrm{d}x\left(1 + \frac{12yy}{aa}\right)$, which will be true for both portions of the bar, whether y be positive or negative; but $y^2 = r^2 - x^2 + d^2 - 2d\sqrt{(r^2 - x^2)}$, and the fluent becomes $x + \frac{12}{aa}\left(r^2x - \frac{1}{3}x^3 + d^2x - 2d\left[r^2 \text{ ARC SINE } \frac{x}{r} - x\sqrt{(r^2 - x^2)}\right]\right)$. When the arc is small, calling the whole versed sine h, we have $y = \frac{1}{3}h - \frac{xx}{2r}$, and $y^2 = \frac{1}{9}h^2 - \frac{hx^2}{3r} + \frac{x^4}{4r^2}$, and the fluent is $x + \frac{12}{aa}\left(\frac{1}{9}h^2x - \frac{hx^3}{9r} + \frac{x^5}{20r^2}\right)$, but when x becomes equal to the semichord c, h being $\frac{cc}{2r}$, the expression becomes $c + \frac{12}{aa}\left(\frac{c^5}{36r^2} - \frac{c^5}{18r^2} + \frac{c^5}{20r^2}\right) = c + \frac{4c^5}{15a^2r^2} = c + \frac{16h^2c}{15a^2}$, which shows the compression or extension in the line of the chord, while c expresses that which the bar would have undergone if it had been straight, and the force had been immediately applied to the axis; the actual change being greater in the proportion of $1 + \frac{16hh}{15aa}$ to 1.

The greatest strain will obviously be at the ends, where the distance from the line of direction of the force is the greatest, the compression or extension of the surface being here to that of the axis, as $a + 6y$ to a (Prop. E.) or as $1 + \frac{4h}{a}$ to 1; consequently the compression or extension in the line of the chord is to the greatest actual change of the substance as $1 + \frac{16hh}{15aa}$ to $1 + \frac{4h}{a}$.

Thus if the depth a were 10 feet, and the height or versed sine $h = 20$, the radius being very large, the whole compression of the chord would be to the whole compression of a similar substance, placed in the direction of the chord, as 5.267 to 1; and the compression at the surface of the ends would be to the compression of the axis there as 9 to 1; and disregarding the insensible obliquity, this compression may be considered as equal throughout the bar; so that the compression at the ends will be to the compression of the chord as 9 to 5.267, or as 17 to 10.

Supposing, for example, such a bar of iron to undergo a change of temperature of 32° of Fahrenheit, which would naturally cause it to expand or contract about $\frac{1}{3000}$ in all its dimensions; then the length of the chord, being limited by the abutments, must now be supposed to be altered $\frac{1}{3000}$ by an external force; and, at the extremities of the abutments, the compression and extension of the metal will amount to about $\frac{1}{3000}$; a change which is equivalent to the pressure of a column of the metal about 3300 feet in height, since M, the height of the modulus of elasticity, is found, for iron and steel, to be about 10,000,000 feet; and such would be the addition to the pressure at one extremity of the abutment, and its diminution at the other, amounting to about five tons for every square inch of the section, which would certainly require some particular precaution, to prevent the destruction of the stones forming the abutment by a force so much greater than they are capable of withstanding without assistance. Should such a case indeed actually occur, it is probable that the extremities would give way a little, and that the principal pressure would necessarily be supported nearer the middle, so that there would be a waste of materials in a situation where they could co-operate but imperfectly in resisting the thrust; an inconvenience which would not occur if the bar were made wider and less deep, especially towards the abutments.

SECTION II.—*Of the Equilibrium of Arches.*

We may now proceed to inquire into the mode of determining the situation and properties of the curve of equilibrium, which represents, for every part of a system of bodies supporting each other, the general direction of their mutual pressure; remembering always that this curve is as much an imaginary line, as the centre of gravity is an imaginary point, the forces being no more actually collected into such a line than the whole weight or inertia of a body is collected in its centre of gravity. Indeed, the situation of the curve is even less definite than that of the centre of gravity, since in many cases it may differ a little according to the nature of the co-operation of the forces which it is supposed to represent. In reality, every gravitating atom entering the structure must be supported by some forces continued in some line, whether regular or irregular, to the fixed points or abutments, and every resisting atom partakes, in a mathematical sense, either positively or negatively, in transmitting a lateral pressure where it is required for supporting any part of the weight: and when we attempt to represent the result of all these collateral pressures by a simple curve, its situation is liable to a slight variation, according to the direction in which we suppose the co-operating forces to be collected. If, for instance, we wished to determine the stability of a joint, formed in a given direction, it would be necessary to consider the magnitude of the forces acting throughout the extent of the joint in a direction perpendicular to its plane, and to collect them into a single result, and it is obvious that the forces, represented by the various elementary curves, may vary very sensibly in their proportion, when we consider their joint operation on a vertical or on an oblique plane; although if the depth of the substance be inconsiderable, this difference will be wholly imperceptible, and in practice it may generally be neglected without inconvenience; calculating the curve upon the supposition of a series of joints in a vertical direction. If, however, we wish to be very minutely accurate, we must attend to the actual direction of the joints in the determination of the curve, and must consider, in the case of a bridge, the whole weight of the structure terminated by a given arch stone, with the materials which it supports, as determining the direction of the curve of equilibrium where it meets the given joint, instead of the weight of the materials terminated by a vertical plane passing through the point of the curve in question, which may sometimes be very sensibly less; this consideration being as necessary for determining the circumstances under which the joints will open, as for the more imaginary possibility of the arch stones sliding upwards or downwards. But we may commonly make a sufficiently accurate compensation for this difference, by supposing the specific gravity of the materials producing the pressure, and the curvature of the line which terminates them, to be a little increased, while the absciss remains equal to that of the curve of equilibrium intersecting the joints.

L. *If two equal parallelepipeds be supported each at one end, and lean against each other at the other, so as to remain horizontal, the curve of equilibrium, representing the general effect of the pressure transmitted through them, will be of a parabolic form.*

The pressure of the blocks, where they meet, will obviously be horizontal, but at the other ends it will be oblique, being the result of this horizontal pressure and of the whole weight of each block. And if we imagine the blocks to be divided into any number of parts, by sections parallel to the ends, which is the only way in which we can easily obtain a regular result, it is evident that the force exerted at any of these sections, by the external portions, must be sufficient to support the lateral thrust and the weight of the internal portions; and its inclination must be such that the horizontal base of the triangle of forces must be to the vertical perpendicular as the lateral thrust to the weight of the internal portion; or, in other words, the lateral thrust remaining constant, the weight supported will be as the tangent of the inclination. But calling the horizontal absciss x, and the vertical ordinate y, the tangent of the inclination will be $\frac{dy}{dx}$; which, in the case of a parallelepiped, must be proportional to the distance x from the contiguous ends; and $x=\frac{mdy}{dx}$; consequently $xdx=mdy$, and $\frac{1}{2}x^2=my$, which is the equation of a parabola. It is usual in such cases to consider the thrusts as rectilinear throughout, and as meeting in the vertical line passing through the centre of gravity of each block; but this mode of representation is evidently only a convenient compendium.

If the blocks were united together in the middle, so as to form a single bar or lever, the forces would be somewhat differently arranged; the upper half of the bar would contain a series of elementary arches, abutting on a series of similar elementary chains in the lower half, so as to take off all lateral thrust from the supports at the ends.

With respect to the transverse strains of levers in general, it may be observed, that the most convenient way of representing them is to consider the axis of the lever as composed of a series of elementary bars, bisected, and crossed at right angles, by as many others extending across the lever, or rather as far as two-thirds of the half depth on each side, where the centre of resistance is situated. The transverse force must then be transmitted unaltered throughout the whole system, acting in contrary directions at the opposite ends of each of the elementary bars constituting the axis; and it must be held in equilibrium, with respect to each of the centres, considered as a fulcrum, by the general result of all the corpuscular forces acting on the longer cross arms; that is, by the difference of the compression or extension on the different sides of the arms. This difference must therefore be constant; and in all such cases the strain or curvature must increase uniformly, and its fluxion must be constant; but if the transverse force be variable, as when the lever supports its own weight, or any further external pressure, the fluxion of the curvature must be proportional to it. Now the transverse force, thus estimated, being the sum of the weights or other forces acting on either side of the given point, the additional weight at the point will be represented by the fluxion of the weight, or by the second fluxion of the strain or curvature, which is ultimately as the fourth fluxion of the ordinate. Also, the fluxion of the strain being as the whole weight on each side, it follows that when the strain is a maximum, and its fluxion vanishes, the whole weight, or the sum of the positive and negative forces on either side, must also vanish; as Mr Dupin has lately demonstrated in a different manner.

M. *In every structure supported by abutments, the tangent of the inclination of the curve of equilibrium to the horizon is proportional to the weight of the parts interposed between the given point and the middle of the structure.*

The truth of this proposition depends on the equality of the horizontal thrust throughout the structure, from which it may be immediately inferred, as in the last proposition. The materials employed for making bridges are not uncommonly such, as to create a certain degree of lateral pressure on the outside of the arch; but as there must be a similar and equal pressure in a contrary direction against the abutment, its effects will be comprehended in the determination of the point at which the curve springs from the abutment, as well as in the direction of the curve itself; so that the circumstance does not afford any exception to the general truth of the law. It is, however, seldom necessary to include the operation of such materials in our calculations, since their lateral pressure has little or no effect at the upper part of the arch, which has the greatest influence on the direction of the curve; and it is also desirable to avoid the unnecessary employment of these soft materials, because they tend to increase the horizontal thrust, and to raise it to a greater height above the foundation of the abutment.

We have therefore generally $\int w\mathrm{d}x = mt = m\frac{\mathrm{d}y}{\mathrm{d}x}$, w being the height of uniform matter, pressing on the arch at the horizontal distance x from the vertex, t the tangent of the inclination of the curve of equilibrium, y its vertical ordinate, and m a quantity proportional to the lateral pressure, or horizontal thrust.

N. *The radius of curvature of the curve of equilibrium is inversely as the load on each part, and directly as the cube of the secant of the angle inclination to the horizon.*

The general expression for the radius of curvature is $r = \frac{(\mathrm{d}z)^3}{\mathrm{d}x\mathrm{dd}y}$; and here, since $m\mathrm{d}y = \mathrm{d}x\int w\mathrm{d}x$, $\mathrm{d}x$ being constant, $m\mathrm{d}^2y = w(\mathrm{d}x)^2$; but $\mathrm{d}z$ being $= \mathrm{d}x\sqrt{(1+t^2)}$, $\frac{(\mathrm{d}z)^2}{\mathrm{dd}y} = \frac{m}{w}(1+t^2)$, and $r = \frac{m}{w}(1+t^2)^{\frac{3}{2}}$; and m being constant, r is inversely as the load w, and directly as the cube of the secant $\sqrt{(1+t^2)}$. The same result may also be obtained from a geometrical consideration of the magnitude of the versed sine of the elementary arc, and the effect of the obliquity of the pressure; the one varying as the square of the secant, the other as the secant simply.

O. *Consequently, if the curve be circular, the load must be everywhere as the cube of the secant.*

P. *If the curve of equilibrium be parabolic, the load must be uniform throughout the span.*

(Prop. L.) The uniformity of the load implies that the superior and inferior terminations of the arch, commonly called the extrados and intrados, should be parallel: but it is not necessary that either of them should be parabolic, unless we wish to keep the curve exactly in the middle of the whole structure. When the height of the load is very great in proportion to that of the arch, the curve must always be nearly parabolic, because the form of the extrados has but little comparative effect on the load at each point.

A parabola will therefore express the general form of the curve of equilibrium in the flat bands of brick or stone, commonly placed over windows and doors, which, notwithstanding their external form, may very properly be denominated flat arches. But if we consider the direction of the joints as perpendicular to the curve, it may easily be shown, from the properties of the wedge, that they must tend to a common axis, in order that the thrust may be equal throughout; and the curve must be perpendicular to them, and consequently circular; but the difference from the parabola will be wholly inconsiderable.

Q. *For a horizontal extrados, and an intrados terminated by the curve itself, which, however, is a supposition merely theoretical, the equation of the curve is* $x = \sqrt{m}\,\mathrm{HL}\frac{y+\sqrt{(yy-aa)}}{a}$.

Since in this case $w = y$ (Prop. M.) we have $\int y\mathrm{d}x = m\frac{\mathrm{d}y}{\mathrm{d}x}$; and $m\mathrm{d}^2y = y\,(\mathrm{d}x)^2$; whence, multiplying both sides by $\mathrm{d}y$, we have $m\mathrm{d}y\mathrm{d}^2y = y\mathrm{d}y(\mathrm{d}x)^2$; and, taking the fluent, $\frac{1}{2}m(\mathrm{d}y)^2 = \frac{1}{2}y^2(\mathrm{d}x)^2$, and $mt^2 = y^2$, which must be corrected by making $y = a$ when t vanishes, so that we shall have $mt^2 = y^2 - a^2$, and $y = \sqrt{(a^2+mt^2)}$. But since $\frac{\mathrm{d}y}{\mathrm{d}x} = t = \sqrt{\left(\frac{yy-aa}{m}\right)}$, $\mathrm{d}x = \mathrm{d}y\sqrt{\frac{m}{(yy-aa)}}$, and $x = \sqrt{m}\,\mathrm{HL}\left(y+\sqrt{[y^2-a^2]}\right) - \sqrt{m}\,\mathrm{HL}a$; whence all the points of the curve may be determined by means of a table of logarithms. But such a calculation is by no means so immediately applicable to practice, as has generally been supposed; for the curve of equilibrium will always be so distant from the intrados at the abutments, as to derange the whole distribution of the forces concerned.

R. *For an arch of equable absolute thickness throughout its length, the equation is* $z = \sqrt{(y^2 - m^2)}$ *and* $x = m\mathrm{HL}\frac{y+\sqrt{(yy-mm)}}{m}$.

The weight of any portion of the half arch being represented by its length z, we have $z = m\frac{\mathrm{d}y}{\mathrm{d}x}$; but $\mathrm{d}z = \mathrm{d}y\sqrt{\left(1+\left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2\right)} = \mathrm{d}y\sqrt{\left(1+\frac{mm}{zz}\right)}$, and $dy = \frac{dz}{\sqrt{\left(1+\frac{mm}{zz}\right)}} = \frac{zdz}{\sqrt{(zz+mm)}}$, of which the fluent is $\sqrt{(z^2+m^2)}$, requiring no further correction than to suppose y initially equal to m; and we have $z = \sqrt{(y^2-m^2)}$. Again, since $dz = dx\sqrt{\left(1+\frac{zz}{mm}\right)}$ we find in the same manner $dx = \frac{mdz}{\sqrt{(mm+zz)}}$, and $x = m\text{HL}\left(z+\sqrt{[mm+zz]}\right) - m\text{HL}m = m\text{HL}\frac{z+y}{m}$. This curve will, therefore, in some cases, be identical with that of the preceding proposition. It is commonly called the catenaria, since it represents the form in which a perfectly flexible chain of equable thickness will hang by its gravity.

S. *If the load on each point of an arch be expressed by the equation $w = a + bx^2$, the equation for the curve of equilibrium will be $my = \frac{1}{2}ax^2 + \frac{1}{12}bx^4$.*

Since the whole load $\int w dx$ is here $ax + \frac{1}{3}bx^3$, we have $m\frac{dy}{dx} = ax + \frac{1}{3}bx^3$, (Prop. M.) and $my = \frac{1}{2}ax^2 + \frac{1}{12}bx^4$.

This expression will, in general, be found sufficiently accurate for calculating the form of the curve of equilibrium in practical cases; and it may easily be made to comprehend the increase of the load from the obliquity of the arch-stones. The ordinate y, at the abutment, being given, the value of m may be deduced from it: and since at the vertex my is simply $\frac{1}{2}ax^2$, the radius of curvature r will here be $\frac{xx}{2y} = \frac{m}{a}$.

T. *If we divide the span of an arch into four equal parts, and add to the weight of one of the middle parts one-sixth of its difference from the weight of one of the extreme parts, we shall have a reduced weight, which will be to the lateral thrust as the height of the arch to half the span, without sensible error.*

The weight of the half arch being expressed by $ax + \frac{1}{3}bx^3$ when x is equal to the whole span, if we substitute x for $\frac{1}{2}x$, it will become $\frac{1}{2}ax + \frac{1}{24}bx^3$, for one of the middle parts, leaving $\frac{1}{2}ax + \frac{7}{24}bx^3$, for the extreme part, which gives $\frac{6}{24}bx^3$ for the difference of the parts, and $\frac{1}{6}$ of this, added to the former quantity makes it $\frac{1}{2}ax + \frac{1}{12}bx^3$: but since $my = \frac{1}{2}ax^2 + \frac{1}{12}bx^4$, dividing by mx, we have $\frac{y}{x} = \frac{\frac{1}{2}ax + \frac{1}{12}bx^3}{m}$.

It is also obvious, that if we subtract, instead of adding, one-sixth of the difference, we have $\frac{1}{2}ax$; and dividing by $\frac{1}{2}x$, we obtain a, and thence $r = \frac{m}{a}$, m being previously found by the proposition.

U. *When the load is terminated by a circular or elliptical arc, $w = a + nb - n\sqrt{(b^2-x^2)}$, and $my = \frac{1}{2}(a+nb)x^2 - \frac{1}{2}nb^2x$ ARC SINE $\frac{x}{b} - \frac{1}{2}nb^2\sqrt{(b^2-x^2)} + \frac{1}{6}n(b^2-x^2)^{\frac{3}{2}} + \frac{1}{3}nb^3$.*

The whole load $\int w dx$ is here $ax + nbx - \frac{1}{2}nb^2$ ARC SINE $\frac{x}{b} - \frac{1}{2}nx\sqrt{(b^2-x^2)}$; and hence $my = \frac{1}{2}ax^2 + \frac{1}{2}nbx^2 - \frac{1}{2}nb^2x$ ARC SINE $\frac{x}{b} + \frac{1}{2}nb^3 - \frac{1}{2}nb^2\sqrt{(b^2-x^2)} + \frac{1}{6}n(b^2-x^2)^{\frac{3}{2}} - \frac{1}{6}nb^3$ (Prop. M.) And the radius of curvature at the vertex will again be $\frac{m}{a}$. When the curve is circular, the axes of the ellipsis being equal, $n = 1$.

If the extrados and intrados are concentric, the calculation requires us to take the difference between the results determining the weight for each curve: but it will commonly be equally accurate in such a case, to consider the depth of the load as uniform, at least when the joints are in the direction of the radii.

X. *The abutment must be higher without than within, by a distance, which is to its breadth, as the horizontal distance of the centre of gravity of the half arch from the middle of the abutment is to the height of the middle of the key-stone above the same point.*

This proposition follows immediately from the proportion of the horizontal thrust to the weight, determined by the property of the lever; the one acting at the distance of the height of the arch from the fulcrum, and the other at the distance of the centre of gravity from the abutment, so as to balance each other; and the oblique direction of the face of the abutment being perpendicular to the thrust compounded of these two forces. The same rule also serves for determining the proper position of the abutment of a beam or rafter of any kind, in order that it may stand securely, without the assistance of friction. But for a bridge, if we calculate the situation of the curve of equilibrium, we obtain the direction of the thrust at its extremity more conveniently, without immediately determining the place of the centre of gravity.

Y. *In order that an arch may stand without friction or cohesion, a curve of equilibrium, perpendicular to all the surfaces of the joints, must be capable of being drawn within the substance of the blocks.*

If the pressure on each joint be not exactly perpendicular to the surfaces, it cannot be resisted without friction, and the parts must slide on each other: this, however, is an event that can never be likely to occur in practice. But if the curve, representing the general pressure on any joint, be directed to a point in its plane beyond the limits of the substance, the joint will open at its remoter end, unless it be secured by the cohesion of the cements, and the structure will either wholly fall, or continue to stand in a new form. (Plate XLII. fig. 5.)

From this condition, together with the determination of the direction of the joints already mentioned (Prop. P.), we may easily find the best arrangement of the joints in a flat arch; the object, in such cases, being to diminish the lateral thrust as much as possible, it is obvious that the common centre of the joints must be brought as near to the arch as is compatible with the condition of the circle remaining within its limits; and it may even happen that the superincumbent materials would prevent the opening of the joints even if the centre were still nearer than this: but if, on the other hand, the arch depended only on its own resistance, and the materials were in any danger of being crushed, it would be necessary to keep the circle at some little distance from its surfaces, even at the expence of somewhat increasing the lateral pressure.

When the curve of equilibrium touches the intrados of an arch of any kind, the compression at the surface must be at least four times as great as if it remained in the middle of the arch-stones (Prop. E.), and still greater than this if the cohesion of the cements is called into action. In this estimate we suppose the transverse sections of the blocks inflexible, so as to cooperate throughout the depth in resisting the pressure on any point; but in reality this cooperation will be confined within much narrower limits, and the diminution of strength will probably be considerably greater than is here supposed, whenever the curve approaches to the intrados of the arch.

The passage of the curve of equilibrium through the middle of each block is all that is necessary to insure the stability of a bridge of moderate dimensions and of sound materials. Its strength is by no means increased, like that of a frame of carpentry, or of a beam resisting a transverse force, by an increase of its depth in preference to any other of its dimensions: a greater depth does, indeed, give it a power of effectually resisting a greater force of external pressure derived from the presence of any occasional load on any part of the structure; but the magnitude of such a load is seldom very considerable, in proportion to the weight of the bridge.

It is of some importance, in these investigations, to endeavour to trace the successive steps by which the fabric of a bridge may commonly be expected to fail. Supposing the materials to be too soft, or the abutments insecure, or any part of the work to be defective, and to afford too little resistance, the length of the curve of the arch being diminished, or its chord extended, it will become flatter, and, consequently, sink; the alteration being by far the greatest, if other things are equal, where the depth is the least, that is, near the crown or key-stone; so that if the curvature was, at first, nearly equal throughout, the crown will sink so much as to cause a rapid increase of curvature on each side in its immediate neighbourhood, which will bring the intrados up to the curve of equilibrium, or even above it, the form of this curve being little altered by the change of that of the arch. The middle remains firm, because the pressure is pretty equally divided throughout the blocks, but the parts newly bent give way to the unequal force, and chip a little at their internal surface; but being reduced in their dimensions by the pressure, they suffer the middle to descend still lower, and are, consequently, carried down with it, so as to be relieved from the inequality of pressure depending on their curvature, and to transfer the effect to the parts immediately beyond them, till these in their turn crumble, and by degrees the whole structure falls. (Plate XLII. fig. 6.)

This explanation will enable us to understand some observations and experiments which the late Professor Robison has related as somewhat paradoxical. He says, that an arch built "of an exceedingly soft and friable stone," the arch-stones being also too short, began to show signs of weakness by the stones chipping about ten feet from the middle, and that it afterwards split at the middle, and fifteen or sixteen feet on each side of it, and also at the abutments. And in some experiments on models of arches in chalk, he found, that "the arch always broke at some place considerably beyond another point, where the first chipping had been observed;" a circumstance which he has not succeeded in sufficiently explaining.

SECTION III.—*Of the Effect of Friction.*

The friction or adhesion of the substances, employed in Architecture, is of the most material consequence, for insuring the stability of the works constructed with them; and it is right that we should know the extent of its operation; it is not, however, often practically necessary to calculate its exact magnitude, because it would seldom be prudent to rely materially on it, the accidental circumstances of agitation or moisture tending very much to diminish its effect. Nor is the cohesion of the cements employed of much further consequence than as enabling them to form a firm connexion, by means of which the blocks may rest more completely on each other than they could do without it; for we must always remember, that we must lose at least half of the strength, before the cohesion of the solid blocks themselves, in the direction of the arch, can be called into action, and at least three fourths before the joints will have any tendency to open throughout their extent.

Z. *The joints of an arch, composed of materials subject to friction, may be situated in any direction lying within the limits of the angle of repose, on either side of the perpendicular to the curve of equilibrium; the angle of repose being equal to the inclination to the horizon at which the materials begin to slide on each other; and the direct friction being to the pressure as the tangent of this angle is to the radius.*

It is obvious, that any other force, as well as that of gravity, will be resisted by the friction or adhesion of the surfaces when its direction is within the limits of the angle at which the substances begin to slide; and it may be inferred from the experiments of Mr Coulomb and Professor Vince, that this angle is constant, whatever the magnitude of the force may be, since the friction is very nearly proportional to the mutual pressure of the substances. The tendency of a body to descend along any plane being as much less than its weight as the height of the plane is less than its length, and the pressure on the plane being as much less than the weight as the length is greater than the horizontal extent, it follows, that, when the weight begins to overcome the friction, the friction must be to the pressure as the height of the plane to its horizontal extent, or as the tangent of the inclination to the radius.

This property of the angle of repose affords a very easy method of ascertaining, by a simple experiment, the friction of the materials employed: taking, for example, a common brick, and placing it, with the shorter side of its end downwards, on another which is gradually raised, we shall find that it will fall over without beginning to slide; and when this happens, the height must be half of the horizontal extent, a brick being twice as long as it is broad: in this case, therefore, the friction must be at least half of the pressure, and the angle of repose at least 30°; and an equilateral wedge of brick could not be forced up by any steady pressure of bricks acting against its sides, in a direction parallel to its base. But the effects of agitation would make such a wedge totally insecure in any practical case; and the determination only serves to assure us, that a very considerable latitude may be allowed to the joints of our materials, when there is any reason for deviating from the proper direction, provided that we be assured of a steady pressure; and much more in brick or stone than in wood, and more in wood than in iron, unless the joints of the iron be secured by some cohesive connexion. It may also be inferred from these considerations, that the direction of the joints can never determine the direction of the curve of equilibrium crossing them, since the friction will always enable them to transmit the thrust in a direction varying very considerably from the perpendicular; although, with respect to any particular joint, of which we wish to ascertain the stability independent of the friction, it would be desirable to collect the result of the elements, of which that curve is the representative, with a proper regard to its direction.

Section IV.—*Earlier Historical Details.*

The original invention of arches, and the date of their general adoption in architecture, have been discussed with great animation by the late Mr King, Mr Dutens, and several other learned antiquaries. Mr King insisted that the use of the arch was not more ancient than the Christian era, and considered its introduction as one of the most remarkable events accompanying that memorable period. Mr Dutens appealed to the structure of the cloacæ, built by the Tarquins, and to the authority of Seneca, who observes, that the arch was generally considered as the invention of Democritus, a Philosopher who lived some centuries before Christ, but that, in his opinion, the simplicity of the principle could not have escaped the rudest architect; and, that long before Democritus, there must have been both bridges and doors, in both of which structures the arch was commonly employed. There do indeed appear to be solitary instances of arches more ancient than the epoch assigned by Mr King to their invention. We find arches concealed in the walls of some of the oldest temples extant at Athens; the cloacæ are said to be arched, not at the opening into the Tiber only, but to a greater distance within it than is likely to have been rebuilt at a later period for ornament; and the fragments of a bridge, still remaining at Rome, bear an inscription which refers its erection to the latter years of the Commonwealth. But it seems highly probable, that almost all the covered ways, constructed in the earlier times of Greece and Rome, were either formed by lintels, like doorways, or by stones overhanging each other, in horizontal strata, and leaving a triangular aperture, or by both these arrangements combined, as is exemplified in the entrance to the treasury of Atreus at Mycenæ, where the lintel has a triangular aperture over it, by which it is relieved from the pressure of the wall above; and this instance serves to show how different the distribution of the pressure on any part of a structure may be, from the simple proportions of the height of the materials above it. Some other old buildings, which have been supposed to be arched, have been found, on further examination, rather to resemble domes, which may be built without centres, and may be left open at the summit, the horizontal curvature producing a transverse pressure, which supports the structure without an ordinary key-stone. And this has been suspected to be the form of the roofs and ceilings of ancient Babylon, where Strabo tells us that the buildings were arched over or "camerated," for the purpose of saving timber: and the bridge of Babylon, which must have been of considerable antiquity, is expressly said, by Herodotus, to have consisted of piers of stone, with a road formed of beams of wood only. It may however be rejoined, that though a dome is not simply an arch, yet it exceeds it in contrivance and mechanical complication; it generally exerts a thrust, and requires either an abutment, or a circular tie; and it is scarcely possible that the inventor of a dome should not have been previously acquainted with the construction of a common arch. Besides the term CAMARA, the Greeks had also PSALIS, APSIS, and THOLUS; the last was particularly appropriate to circular domes; but the variety of appellations seems to prove that the thing must have been perfectly familiar; and the term PSALIS is supposed to have been applied from the appearance of the wedged arch-stones, viewed in their elevation, which could not have been observable in a dome of any kind.

From these outlines of the origin of the art of building bridges, we may pass on rapidly to the latest improvements which have been made, in Great Britain, and on the Continent, in the practice of this department of architecture. A very ample detail of the most important operations, that are generally required to be performed in it, may be found in the numerous Reports of the ingenious Mr Smeaton, published since his death by the Society of Civil Engineers in London. They contain a body of information comprehending almost every case that can occur to a workman, in the execution of such structures; and even where they have to record an accidental failure, the instruction they afford is not less valuable than where the success has been more complete.

Respecting the general arrangement of a bridge, and the number of arches to be employed, in the case of a wide river, Mr Smeaton has expressed his approbation of a few wide and flat arches, supported by good abutments, in preference to more numerous piers, which unnecessarily interrupt the water-way. In a case where a long series of small arches was required, he has made them so flat, and the piers so slight, that a single pier would be incapable of withstanding the thrust of its arch: but in order to avoid the destruction of the whole fabric in case of an accident, he has intermixed a number of stronger piers, at certain intervals, among the weaker ones. Where several arches, of different heights, were required, he commonly recommended different portions of the same circle for all of them; a mode which rendered the lateral thrust nearly equal throughout the fabric, and had the advantage of allowing the same centre to be employed for all, with some little addition at the ends to adapt it to the larger arches. He records the case of Old Walton bridge, in which the wooden superstructure had sunk two feet, so as to become part of a circle 700 feet in diameter, and the thrust, thus increased, had forced the piers considerably out of their original situation: a striking proof that the principles of the pressure of arches must not be neglected, even when frames of carpentry are concerned.

Mr Smeaton particularly describes the inconveniences arising from the old method of laying the foundations of piers, which was introduced soon after the Conquest, and which is particularly exemplified in London Bridge. The masonry commences above low water mark, being supported on piles, which would be exposed to the destructive alternation of moisture and dryness, with the access of air, if they were not defended by other piles, forming projections partly filled with stone, and denominated sterlings; which, in their turn, occasionally require the support and defence of new piles surrounding them, since they are not easily removed when they decay; so that, by degrees, a great interruption is occasioned by the breadth of the piers, thus augmented, requiring, for the transmission of the water, an increase of velocity, which is not only inconvenient to the navigation, but also carries away the bed of the river under the arches, and immediately below the bridge, making deep pools or excavations, which require from time to time to be filled up with rubble stones; while the materials, which have been carried away by the stream, are deposited a little lower down in shoals, and very much interfere with the navigation of the river. From these circumstances, as well as from the effects of time and decay, it has happened, according to late reports, that the repairs of London Bridge have often amounted, for many years together, to L. 4000 a year, while those of Westminster and Blackfriars Bridges have not cost so many hundreds. It is true, that the fall produces a trifling advantage in enabling the London water-works to employ more of the force of the tide in raising water for the use of the city; and this right, being established as a legal privilege, has long delayed the improvements, which might otherwise have been attempted, for the benefit of the navigation of the river. The interest of the proprietors of the water-works has been valued at L. 125,000; and it has been estimated that L. 50,000 would be required for the erection of steam-engines to supply their place; while, on the other hand, it is said that from thirty to forty persons, on an average, have perished annually from the dangers of the fall under the bridge. (Plate XLIII. fig. 7, 8.)

But Mr Smeaton, as well as his predecessor Mr Labelye, appears sometimes to have gone into a contrary extreme, and to have been somewhat too sparing in the use of piles. It is well known that the opening of Westminster Bridge was delayed for two years on account of the failure of a pier, the foundation of which had been partly undermined by the incautious removal of gravel from the bed of the river, in its immediate neighbourhood; a circumstance which would scarcely have occurred if piles had been more freely employed in securing the foundation. The omission, however, did not arise from a want of a just estimate of the importance of piles in a loose bottom, but from a confidence, founded on examination as the work advanced, that the bed of the river was already sufficiently firm. Mr Smeaton directed the foundations of Hexham Bridge to be laid, as those of Westminster Bridge had been, by means of caissons, or boxes, made water-tight, and containing the bottom of the pier, completed in masonry well connected together, and ready to be deposited in its proper place by lowering the caissons, and then detaching the sides, which are raised for further use, from the bottoms, which remain fixed as a part of the foundations immediately resting on the bed of the river, previously made smooth for their reception, and sometimes also rendered more firm by piles and a grating of timber. By a careful examination of the bottom of the river at Hexham, Mr Smeaton thought he had ascertained that the stratum of gravel, of which it consisted, was extremely thin, and supported by a quicksand, much too loose to give a firm hold to piles, while he supposed the gravel strong enough to bear the weight of the pier, if built in a caisson. The bridge was a handsome edifice, with elliptical arches, and stood well for a few years; but an extraordinary flood occurred, which caused the water to rise five feet higher above than below the bridge, and to flow through it with so great a velocity, as to undermine the piers, and cause the bridge to divide longitudinally, and fall in against the stream; a circumstance so much the more mortifying to the eminent engineer who had constructed it, as it was the only one of his works that, " in a period of thirty years," had been known to fail. It was observed that some of the piers, which had been built in coffer-dams, with the assistance of some piles, withstood the violence of the flood; and it is remarkable, that the whole bridge has been rebuilt by a provincial architect with perfect success, having stood without any accident for many years.

It seems, therefore, scarcely prudent to trust any very heavy bridge to a foundation not secured by piles, unless the ground on which it stands is an absolute rock; and in this case, as well as when piles are to be driven and sawed off, it is generally necessary to have recourse to a coffer-dam. In the instance of the bridge at Harraton, for example, where the rock is nine feet below the bed of the river, Mr Smeaton directs that the piles forming the coffer-dam be rebated into each other, driven down to the rock, and secured by internal stretchers, before the water contained within them is pumped out. In some cases, a double row of piles, with clay between them, has been employed for forming a coffer-dam; but in others it has been found more convenient to drive and cut off the piles under water, by means of proper machinery, without the assistance of a coffer-dam.

Piles are employed of various lengths, from 7 to 16 feet or more, and from 8 to 10 inches in thickness, and they are commonly shod with iron. Smeaton directs them to be driven till it requires from 20 to 40 strokes of the pile driver to sink them an inch, according to the magnitude of the weight, and the firmness required in the work. He was in the habit of frequently recommending the piles surrounding the piers to be secured by throwing in rubble stone, so as to form an inclined surface, sloping gradually from the bridge upwards and downwards. In the case of Coldstream Bridge, it was also found necessary to have a partial dam, or artificial shoal, thrown across the river a little below the bridge, in order to lessen the velocity of the water, which was cutting up the gravel from the base of the piles. But all these expedients are attended with considerable inconvenience, and it is better to avoid them in the first instance by leaving the water-way as wide and as deep as possible, and by making the foundations as firm and extensive as the circumstances may require.

The angles of the piers, both above and under water, are commonly rounded off, in order to facilitate the passage of the stream, and to be less liable to accidental injury. Mr Smeaton recommends a cylindrical surface of 60° as a proper termination; and two such surfaces, meeting each other in an angle, will approach to the outline of the head of a ship, which is calculated to afford the least resistance to the water gliding by it.

We find that, in the year 1769, the earth, employed for filling up the space between the walls of the North Bridge in Edinburgh, had forced them out, so as to require the assistance of transverse bars and buttresses for their support. In the more modern bridges, these accidents are prevented by the employment of longitudinal walls for filling up the haunches, with flat stones covering the intervals between them, instead of the earth, or the more solid materials which were formerly used, and which produced a greater pressure both on the arch and on the abutments, as well as a transverse thrust against the side walls. For the inclination of the road passing over this bridge, Mr Smeaton thought a slope of 1 in 12 not too great; observing that horses cannot trot even when the ascent is much more gradual than this, and that if they walk, they can draw a carriage up such a road as this without difficulty: and, indeed, the bridge at Newcastle appears, for a short distance, to have been much steeper. But it has been more lately argued, on another occasion, that it is a great inconvenience in a crowded city, to have to lock the wheel of a loaded waggon; that this is necessary at all times on Holborn Hill, where the slope is only 1 in 18; while in frosty weather this street is absolutely impassable for such carriages: and the descent of Ludgate Hill, which is only 1 in 36, is considered as much more desirable, when it is possible to construct a bridge with an acclivity so gentle.

Section V.—*Improvements of the Port of London.*

From the study of Mr Smeaton's diversified labours, we proceed to take a cursory view of the Parliamentary Inquiry respecting the improvement of the Port of London, which has brought forwards a variety of important information, and suggested a multiplicity of ingenious designs. The principal part of that which relates to our present subject is contained in the Second and Third Reports from the Select Committee of the House of Commons, on the improvement of the Port of London; ordered to be printed 11th July 1799, and 28th July 1800.

We find in these Reports some interesting details respecting the history of London Bridge, which appears to have been begun, not, as Hume tells us, by William Rufus, who was killed in 1100, but in 1176, under Henry II.; and to have been completed in 33 years. The piles are principally of elm, and they have remained for six centuries without material decay; although a part of the bridge fell, and was rebuilt about 100 years after it was begun. Rochester, York, and Newcastle Bridges were also built in the twelfth century, as well as the Bridge of St Esprit at Avignon. About 50 years ago, the middle pier of London Bridge was removed; the piles were drawn by a very powerful screw, commonly used for lifting the wheels of the water-works; and a single arch was made to occupy the place of two. In consequence of this, the fall was somewhat diminished, and it was necessary partially to obstruct the channel again, in order that the stream should have force enough for the water-works; but it was very difficult to secure the bottom from the effects of the increased velocity under the arch. Several strong beams were firmly fixed across the bed of the river, but only two of them retained their situations for any length of time; and the materials carried away had been deposited below the middle arch, so as to form a shoal, which was only 16 inches below the surface at low water. The Reports contain also much particular information respecting Blackfriars Bridge, the piles for which were driven under water, and cut off level with the bed of the foundations, by a machine of Mr Mylne's invention. The expense of Blackfriars Bridge, including the purchase of premises, was about L. 260,000; that of the building only was L. 170,000. Westminster Bridge, built in the beginning of the century, cost about L. 400,000.

The committee had received an immense variety of plans and proposals for docks, wharfs, and bridges, and many of these have been published in the Reports, together with engraved details on a very ample scale. They finally adopted three resolutions respecting the rebuilding of London Bridge.

"1. That it is the opinion of this Committee, that it is essential to the improvement and accommodation of the port of London, that London Bridge should be rebuilt upon such a construction as to permit a free passage, at all times of the tide, for ships of such a tonnage, at least, as the depth of the river would admit of, at present, between London Bridge and Blackfriars Bridge.

"2. That it is the opinion of this Committee, that an iron bridge, having its centre arch not less than 65 feet high in the clear, above high-water-mark, will answer the intended purpose, and at the least expense.

"3. That it is the opinion of this Committee, that the most convenient situation for the New Bridge, will be immediately above St Saviour's Church, and upon a line from thence to the Royal Exchange."

In a subsequent Report, ordered to be printed 3d June 1801, we find a plan for a magnificent iron bridge of 600 feet span, which had been submitted to the Committee by Messrs Telford and Douglas. Mr Telford's reputation in his profession as an engineer deservedly attracted the attention of the Committee; but many practical difficulties having been suggested to them, they circulated a number of queries relating to the proposal, among such persons of science, and professional architects, as were the most likely to have afforded them satisfactory information. But the results of these inquiries are not a little humiliating to the admirers of abstract reasoning and of geometrical evidence; and it would be difficult to find a greater discordance in the most heterodox professions of faith, or in the most capricious variations of taste, than is exhibited in the responses of our most celebrated professors, on almost every point submitted to their consideration. It would be useless to dwell on the numerous errors with which many of the answers abound; but the questions will afford us a very convenient clue for directing our attention to such subjects of deliberation as are really likely to occur in a multiplicity of cases; and it will perhaps be possible to find such answers for all of them, as will tend to remove the greater number of the difficulties which have hitherto embarrassed the subject.

Questions respecting the construction of a cast iron bridge, of a single arch, 600 feet in the span, and 65 feet rise. (Plate XLII. fig. 7.)

1. *What parts of the bridge should be considered as wedges, which act on each other by gravity and pressure, and what parts as weight, acting by gravity only, similar to the walls and other loading, usually erected upon the arches of stone bridges. Or does the whole act as one frame of iron, which can only be destroyed by crushing its parts?*

The distribution of the resistance of a bridge may be considered as in some measure optional, since it may be transferred from one part of the structure to another, by wedging together most firmly those parts which we wish to be most materially concerned in it. But there is also a natural principle of adjustment, by which the resistance has a tendency to be thrown where it can best be supported; for the materials being always more or less compressible, a very small change of form, supposed to be equal throughout the structure, will relieve those parts most which are the most strained, and the accommodation will be still more effectual when the parts most strained undergo the greatest change of form. Thus, if the flatter ribs, seen at the upper part of the proposed structure, supported any material part of its weight, they would undergo a considerable longitudinal compression, and being shortened a little, would naturally descend very rapidly upon the more curved, and consequently stronger parts below, which would soon relieve them from the load improperly allotted to them; the abutment would also give way a little, and be forced out, by the greater pressure at its upper part, while the lower part remained almost entirely unchanged.

It is, however, highly important that the work should, in the first instance, be so arranged as best to fulfil the intended purposes, and especially that such parts should have to support the weight as are able to do it with the least expense of lateral thrust, which is the great evil to be dreaded in a work of these gigantic dimensions, the materials themselves being scarcely ever crushed, when the arch is of a proper form; and the failure of an iron bridge, by the want of ultimate resistance of its parts to a compressing force, being a thing altogether out of our contemplation; and it is obvious that the greater the curvature of the resisting parts, the smaller will be the lateral thrust on the abutments.

We may, therefore, sufficiently answer this question, by saying, that the whole frame of the proposed bridge, so far as it lies in or near the longitudinal direction of the arch, may occasionally cooperate in affording a partial resistance if required; but that the principal part of the force ought to be concentrated in the lower ribs, not far remote from the intrados.

But it is by no means allowable to calculate upon a curve of equilibrium exactly coinciding with the intrados; since, if this supposition were realized, we should lose more than three-fourths of the strength of our materials, and all the stability of the joints independent of cohesion, so that the slightest external force might throw the curve beyond the limits of the joint, and cause it to open. Nor can we always consider the curve of equilibrium as parallel to the intrados: taking, for example, the case of a bridge like Blackfriars, the curve of equilibrium, passing near the middle of the arch-stones, is, and ought to be, nine or ten feet above the intrados at the abutment, and only two or three feet at the crown; so that the ordinates of this curve are altogether different from the ordinates which have hitherto been considered by theoretical writers. It may be imagined that this difference is of no great importance in practice; but its amount is much greater than the difference between the theoretical curves of equilibrium, determined by calculation, and the commonest circular or elliptical arches.

With respect to the alternative of comparing the bridge with masonry or with carpentry, we may say, that the principles on which the equilibrium of bridges is calculated, are altogether elementary, and independent of any figurative expressions of strains and mechanical purchase, which are employed in considering many of the arrangements of carpentry, and which may indeed, when they are accurately analysed, be resolved into forces opposed and combined in the same manner as the thrusts of a bridge. It is, therefore, wholly unnecessary, when we inquire into the strength of such a fabric, to distinguish the thrusts of masonry from the strains of carpentry, the laws which govern them being not only similar but identical; except that a strain is commonly understood as implying an exertion of cohesive force, and we have seen that a cohesive force ought never to be called into action in a bridge, since it implies a great and unnecessary sacrifice of the strength of the materials employed. If, indeed, we wanted to cross a mere ditch, without depending on the firmness of the bank, we might easily find a beam of wood or a bar of iron strong enough to afford a passage over it, unsupported by any abutment, because, in a substance of inconsiderable length, we are sure of having more strength than we require. But to assert that an iron bridge of 600 feet span "is a lever exerting a vertical pressure only on the abutments," is to pronounce a sentence from the lofty tribunal of refined science, which the simplest workman must feel to be erroneous. But, in this instance, the error is not so much in the comparison with the lever, as in the inattention to the mode of fixing it: for a lever or beam of the dimensions of the proposed bridge, lying loosely on its abutments, would probably be at least a hundred times weaker than if it were firmly connected with the abutments as a bridge is, so as to be fixed in a determinate direction. And the true reason of the utility of cast iron for building bridges, consists not, as has often been supposed, in its capability of being united so as to act like a frame of carpentry, but in the great resistance which it seems to afford to any force tending to crush it.

QUESTION II. *Whether the strength of the arch is affected, and in what manner, by the proposed increase of its width towards the two extremities or abutments, when considered vertically and horizontally. And if so, what form should the bridge gradually acquire?*

The only material advantage, derived from widening the bridge at the ends, consists in the firmness of the abutments; and this advantage is greatly diminished by the increase of horizontal thrust which is occasioned by the increase of breadth; while the curve of equilibrium is caused to deviate greatly from a circular or parabolic arc, in consequence of the great inequality of the load on the different parts; and there seems to be no great difficulty in forming a firm connexion between a narrow bridge and a wider abutment, without this inconvenience. The lateral strength of the fabric, in resisting any horizontal force, would be amply sufficient, without the dilatation at the ends. Perhaps the form was suggested to the inventor by the recollection of the partial failure of an earlier work of the same kind, which has been found to deviate considerably from the vertical plane in which it was originally situated: but in this instance there seems, if we judge from the engravings which have been published, to have been a total deficiency of oblique braces; and the abutments appear to have been somewhat less firm than could have been desired, since one of them contains an arch and some warehouses, instead of being composed of more solid masonry. (Plate XLIII. fig. 9.)

QUESTION III. *In what proportions should the weight be distributed from the centre to the abutments, to make the arch uniformly strong?*

This question is so comprehensive, that a complete answer to it would involve the whole theory of bridges; and it will be necessary to limit our investigations to an inquiry whether the structure, represented in the plan, is actually such as to afford a uniform strength, or whether any alterations can be made in it, compatible with the general outlines of the proposal, to remedy any imperfections which may be discoverable, in the arrangement of the pressure.

There is an oversight in some of the official answers to this question, from quarters of the very first respectability, which requires our particular attention. The weight of the different parts of the bridge has been supposed to differ so materially from that which is required for producing an equilibrium in a circular arch of equable curvature, that it has been thought impossible to apply the principles of the theory in any manner to an arch so constituted, at the same time that the structure is admitted to be tolerably well calculated to stand, when considered as a frame of carpentry. The truth is, that it is by no means absolutely necessary, nor often perfectly practicable, that the mean curve of equilibrium should agree precisely in its form with the curves limiting the external surfaces of the parts bearing the pressure, especially when they are sufficiently extensive to admit of considerable latitude within the limits of their substance. It may happen in many cases, that the curve of equilibrium is much flatter in one part, and more convex in another, than the circle which approaches nearest to it; and yet the distance of the two curves may be inconsiderable, in comparison with the thickness of the parts capable of cooperating in the resistance. The great problem, therefore, in all such cases, is, to determine the precise situation of the curve of equilibrium in the actual state of the bridge; and when this has been done, the directions of the ribs, in the case of an iron bridge, and of the joints of the arch-stones, in a stone bridge, may be so regulated as to afford the greatest possible security; and if this security is not deemed sufficient, the whole arrangement must be altered.

Considering the effect of the dilatation at the ends in increasing the load, we may estimate the depth of the materials causing the pressure at the abutments as about three times as great as at the crown; the plan not being sufficiently minute to afford us a more precise determination; and it will be quite accurate enough to take $w = a + bx^2$ (Prop. S.) for the load, w becoming $= 3a$ when x is 300 feet, whence $90,000\, b = 2a$, and $b = \frac{1}{45,000}a$; we have then $my = \frac{1}{2}ax^2 + \frac{1}{540,000}ax^4$ for the value of the ordinate. Now the obliquity to the horizon being inconsiderable, this ordinate will not ultimately be much less than the whole height of the arch; and its greatest value may be called 64 feet; consequently when $x = 300$, we have $64\, m = \frac{1}{2}\, a \times 90,000 + \frac{1}{6}\, a \times 90,000$, and the radius of curvature at the vertex $r = \frac{m}{a} = 937.5$ feet, while the radius of the intrados is 725 feet, and that of a circle passing through both ends of the curve of equilibrium, as we have supposed them to be situated, 735 feet. Hence, y being $= \frac{1}{1875}x^2\left(1 + \frac{1}{270000}x^2\right)$, we may calculate the ordinates at different points, and compare them with those of the circular curves.

Distance x.	Versed sine of the intrados.	Versed sine of the circular arc.	Ordinate y.
50	1.73	1.71	1.34
100	6.94	6.82	5.38
150	15.66	15.43	13.00
200	28.13	27.70	24.50
250	44.42	43.81	41.01
300	65.00	64.00	64.00

Hence it appears that, at the distance of 200 feet from the middle, the curve of equilibrium will rise more than 3 feet above its proper place; requiring a great proportion of the pressure to be transferred to the upper ribs, with a considerable loss of strength, for want of a communication approaching more nearly to the direction of the curve. If we chose to form the lower part of the structure of two series of frames, each about 4 feet deep, with diagonal braces, we might provide amply for such an irregularity in the distribution of the pressure; but it would be necessary to cast the diagonals as strong as the blocks, in order to avoid the inequality of tension from unequal cooling, which is often a cause of dangerous accidents; it would, however, be much better to have the arch somewhat elliptical in its form, if the load were of necessity such as has been supposed.

Question 4. *What pressure will each part of the bridge receive, supposing it divided into any given number of equal sections, the weight of the middle section being given. And on what parts, and with what force, will the whole act upon the abutments?*

It appears from the preceding calculations, that the weight of the "middle section" alone is not sufficient for determining the pressure in any part of the fabric; although, when the form of the curve of equilibrium has been found, its radius of curvature at the summit must give at once the length of a similar load equivalent to the lateral thrust; and by combining this thrust with the weight, or with the direction of the curve, the oblique thrust at any part of the arch may be readily found. Thus, since at the abutment $w = a + bx^2 = 3a$, and $bx^2 = 2a$, we have $y = \frac{1}{2}\frac{a}{m}x^2 + \frac{1}{12}\frac{b}{m}\frac{x^4}{}$, and $\frac{dy}{dx}$, the tangent of the inclination, becomes $= \frac{a}{m}x + \frac{1}{3}\frac{b}{m}x^3 = \frac{ax}{m} + \frac{2}{3}\frac{ax}{m} = \frac{5}{3}\frac{x}{r} = \frac{5}{3}\cdot\frac{300}{937.5} = \frac{8}{15} = .5333$; consequently the horizontal thrust will be to the weight of the half arch as 15 to 8, and to that of the whole arch as 15 to 16. Now the arch is supposed to contain 6500 tons of cast iron, and together with the road, will amount, according to Professor Robison's estimate, to 10,100 tons; so that the lateral thrust on each abutment is 9470 tons; and since this is equal to the weight of 937.5 feet in length, of the thickness of the crown, the load there must be about 10 tons for each foot of the length. Hence, it appears, that although the thrust, thus calculated, is greater than the weight of a portion of equal length with the apparent radius at the crown, it is less than would be inferred from the angular direction of the intrados at the abutment: the inclination of the termination of the arch being 24° 27′, while that of the true curve of equilibrium is 28° 4′; that is, about one-tenth greater.

As a further illustration of the utility of this mode of computation, we may take the example of an arch of Blackfriars Bridge. The radius of curvature, as far as four-fifths of the breadth, is here 56 feet; and we may suppose, without sensible error, the whole load to be that which would be determined by the continuation of the same curve throughout the breadth. Now, the middle of the arch stones, at the distance of 50 feet from the middle of the bridge, that is, immediately over the termination of the abutment, is about 12 feet above that termination, and at the crown about three feet above the intrados, so that we have only 31 feet for the extreme value of y, while the whole height of the arch is 40; and a being 6.58 feet, we find (Prop. U.) $my = 13,510 = 31m$, whence $m = 436$, and $\frac{m}{a} = r = 66\frac{1}{4}$; we also obtain the values of the ordinates of the curve as in the annexed table.

Distance x.		Ordinate y.		Middle of the Arch-Stones.
10 FEET	-	.76	-	.90
20	-	3.12	-	3.72
25	-	5.13	-	6.12
30	-	7.71	-	8.75
40	-	15.81	-	16.81
50	-	31.00	-	31.00

Hence it appears that the greatest deviation is about 30 feet from the middle, where it amounts to a little more than a foot. But if we suppose this deviation divided by a partial displacement of the curve at its extremities, as it would probably be in reality, even if the resistance were confined to the arch-stones, it would be only about half as great in all three places; and even this deviation will reduce the strength of the stones to two-thirds, leaving them however still many times stronger than can ever be necessary. The participation of the whole fabric, in supporting a share of the oblique thrust, might make the pressure on the arch-stones somewhat less unequal, and the diminution of their strength less considerable; but it would be better that the pressure should be confined almost entirely to the arch-stones, as tending less to increase the horizontal thrust, which is here compressed by $m = 436$, implying the weight of so many square feet of the longitudinal section of the bridge; while, if we determined it from the curvature of the intrados, it would appear to be only $56a = 368$.

In this calculation, the oblique direction of the joints, as affecting the load, has not been considered; but its effect may be estimated by merely supposing the specific gravity of the materials to be somewhat increased. Thus, since the back of each arch-stone is about one-eighth wider than its lower end, the weight of the materials pressing on it will be about one-sixteenth greater than would press on it, if it were of uniform thickness; and this increase will be very nearly proportional to w, the whole load at each part; so that it will only affect the total magnitude of the thrust, which, instead of 436, must be supposed to amount to about 463. If also great accuracy were required, it would be necessary to appreciate the different specific gravities of the various materials constituting the load; since they are not altogether homogeneous; but so minute a calculation is not necessary in order to show the general distribution of the forces concerned, and the sufficiency of the arrangement for answering all the purposes intended.

QUESTION 5. *What additional weight will the bridge sustain, and what will be the effect of a given weight placed upon any of the before mentioned sections?*

When a weight is placed on any part of a bridge, the curve of equilibrium must change its situation more or less, according to the magnitude of the weight; and the tangent of its inclination must now be increased by a quantity proportional to the additional pressure to be supported, which, if the weight were placed in the middle of the arch, would always be equal to half of it; but when the weight is placed at any other part of the arch, if we find the point where the whole thrust is horizontal, the vertical pressure to be supported at each point of the curve must obviously be equal to the weight of the materials interposed between it and this new summit of the curve. Now, in order to find where the thrust is horizontal, we must divide the arch into two such portions, that their difference, acting at the end of a lever of the length of half the span, that is, of the distance from the abutment, may be equivalent to the given weight, acting on a lever equal to its distance from the other abutment, to which it is nearest; consequently this difference must be to the weight as the distance of the weight from the end to half the span; and the distance of the new summit of the curve from the middle must be such, that the weight of materials intercepted between it and the middle shall be to the weight as the distance of the weight from the end to the whole span; and the tangent of the inclination must everywhere be increased or diminished by the tangent of the angle at which the lateral thrust would support the weight of this portion of the materials; except immediately under the weight, where the two portions of the curve will meet in a finite angle, at least if we suppose the weight to be collected in a single point.

If, for example, a weight of 100 tons, equal to that of about 10 feet of the crown of the arch, be placed half-way between the abutment and the middle; then the vertex of the curve, where the thrust is horizontal, will be removed $2\frac{1}{2}$ feet towards the weight; but the radius being 937.5 feet, the tangent of the additional inclination will be $\frac{2.5}{937.5} = \frac{1}{375}$, and each ordinate of the curve will be increased $\frac{1}{375}$ of the absciss, reckoning from the place of the weight to the remoter abutment; but between the weight and the nearest abutment, the additional pressure at each point will be $10 - 2.5 = 7.5$ feet, consequently the tangent will be $\frac{1}{125}$, and the additions to the ordinates at the abutments will be $\frac{450}{375}$ and $\frac{150}{125}$, each equal to $1\frac{1}{5}$ foot, and at the summit $\frac{150}{375} = \frac{2}{5}$, which, being deducted, the true addition to the height of the curve will appear to be $\frac{4}{5}$. But the actual height will remain unaltered, since the curve is still supposed to be terminated by the abutments, and to pass through the middle of the key-stone; and we have only to reduce all the ordinates in the proportion of 64.8 to 64. Thus, at 200 feet from the summit, the ordinate, instead of $24.50 + \frac{200}{375} = 25.03$, will be 24.72, so that the curve will be brought $2\frac{1}{2}$ inches nearer to the intrados, which, in the proposed fabric, would by no means diminish its strength; while, on the opposite side, immediately under the weight, the ordinate $13 - \frac{150}{375} = 12.6$ will be reduced to 12.45, and the curve raised between six and seven inches, which is a change by no means to be neglected in considering the resistances required from each part of the structure. We ought also, if great accuracy were required, to determine the effect of such a weight in increasing the lateral thrust, which would affect in a slight degree the result of the calculation; but it would not amount, in the case proposed, to more than one-eightieth of the whole thrust.

It is obvious that the tendency of any additional weight, placed near the middle of a bridge, is to straighten the two branches of the curve of equilibrium, and that, if it were supposed infinite, it would convert them into right lines; provided, therefore, that such right lines could be drawn without coming too near the intrados at the haunches, the bridge would be in no danger of giving way, unless either the materials were crushed, or the abutments were forced out. In fact, any bridge well constructed might support a load at least equal to its own weight, with less loss of strength than would arise from some such errors, as have not very uncommonly been committed, even in works which have on the whole succeeded tolerably well.

Question 6. *Supposing the bridge executed in the best manner, What horizontal force will it require, when applied to any particular part, to overturn it, or press it out of the vertical plane?*

If the bridge be well tied together, it may be considered as a single mass, standing on its abutments; its mean breadth being about 80 feet, and its weight 10,100 tons; and such a mass would require a lateral pressure at the crown of the arch of about 7000 tons to overset it. Any strength of attachment to the abutments would, of course, make it still firmer, and any want of connexion between the parts weaker; and since the actual resistance to such a force must depend entirely on the strength of the oblique connexion between the ribs, it is not easy to define its magnitude with accuracy: but, as Professor Robison has justly remarked, the strength would be increased by causing the braces to extend across the whole breadth of the half arch. The single ribs, if wholly unconnected, might be overset by an inconsiderable force, since they stand in a kind of tottering equilibrium; and something like this appears to have happened to the bridge at Wearmouth. Dr Hutton, indeed, mentions some "diagonal iron bars" in this bridge; but these were perhaps added after its first erection, to obviate the "twisting," which had become apparent, since they are neither exhibited in the large plates of the bridge, nor mentioned in the specification of the patent.

Question 7. *Supposing the span of the arch to remain the same, and to spring ten feet lower, What additional strength would it give the bridge? Or, making the strength the same, What saving may be made in the materials? Or, if, instead of a circular arch, as in the plates and drawings, the bridge should be made in the form of an elliptical arch, What would be the difference in effect, as to strength, duration, convenience, and expenses?*

The question seems to suppose the weight of the materials to remain unaltered, and the parts of the structure, that would be expanded, to be made proportionally lighter; which could not be exactly true, though there might be a compensation in some other parts. Granting, however, the weight to be the same under both circumstances, if the ordinate y at the end be increased in the proportion of 64 to about 73, the curvature at the vertex will be increased, and the lateral thrust diminished in the same ratio, the 9470 tons being reduced to 8300. The additional thrust occasioned by any foreign weight would also be lessened, but not the vertical displacement of the curve derived from its pressure; and since the whole fabric might safely be made somewhat lighter, the lightness would again diminish the strain. The very least resistance that can be attributed to a square inch of the section of a block of cast iron, is about 50 tons, or somewhat more than 100,000 pounds. It is said, indeed, that Mr William Reynolds found, by accurate experiments, that 400 tons were required, to crush a cube of a quarter of an inch, of the kind of cast iron called gun-metal, which is equivalent to 6400 tons for a square inch of the section. But this result so far exceeds any thing that could be expected, either from experiment or from analogy, that it would be imprudent to place much reliance on it in practice; the strength attributed to the metal being equivalent to the pressure of a column 2,280,000 feet in height, which would compress it to about four-fifths of its length, since the height of the modulus of elasticity (Prop. G.) is about 10,000,000 feet. The greatest cohesive force, that has ever been observed in iron or steel, does not exceed 70 tons for a square inch of the section, and the repulsive force of a homogeneous substance has not been found, in any other instance, to be many times greater or less than the cohesive. There cannot, however, be any doubt that the oblique thrust, which amounts to 10,730 tons, would be sufficiently resisted by a section of 215 square inches, or, if we allowed a load amounting to about one-third only of the whole strength, by a section of 600 square inches; and since each foot of an iron-bar, an inch square, weighs three pounds, and the whole length of the arch nearly a ton, the 600 square inches would require nearly as many tons to be employed in the ribs affording the resistance, upon this very low estimate of the strength of cast iron. The doubts here expressed respecting Mr Reynolds's results, have been fully justified by some hasty experiments, which have been obligingly made by the son of a distinguished architect: he found that two parallelepipeds of cast iron, one eighth of an inch square, and a quarter of an inch long, were crushed by a force of little more than a ton. The experiments were made in a vice, and required considerable reductions for the friction. The mode of calculation may deserve to be explained, on account of its utility on other similar occasions. Supposing the friction to be to the pressure

11

on the screw as 1 to m, and the pressure on the screw to the actual pressure on the substance as n to 1, calling this pressure x, the pressure on the screw will be nx, and the friction $\frac{nx}{m}$; but this resistance will take from the gross ultimate pressure f a force which is to the friction itself, as the velocity of the parts sliding on each other is to the velocity of the part producing the ultimate pressure, a proportion which we may call p to 1; and the force remaining will be the actual pressure; that is, $f - \frac{pnx}{m} = x$, and $x = \frac{m}{m+pn}f$. In these experiments, the gross force f, as supposed to be exerted on the iron, was 4 tons; the friction $\frac{1}{m}$, was probably about $\frac{1}{4}$, the screw not having been lately oiled; the distance of the screw from the centre of motion was to the length of the whole vice as 3 to 4, whence n was $\frac{4}{3}$, and p was 8.44, the middle of the screw describing 4.22 inches, while the cheek of the vice moved through $\frac{1}{2}$ an inch: consequently $\frac{m}{m+pn}$ was $\frac{4}{4+11.25} = \frac{1}{3.81}$, and the corrected pressure becomes $\frac{4}{3.81}$. In several experiments made with still greater care, and with an improved apparatus of levers, the mean force required to crush a cube of a quarter of an inch was not quite $4\frac{1}{2}$ tons, instead of 400.

Calcareous freestone supports about a ton on a square inch, which is equal to the weight of a column not quite 2000 feet in height: consequently an arch of such freestone, of 2000 feet radius, would be crushed by its own weight only, without any further load; and for an arch like that of a bridge, which has other materials to support, 200 feet is the utmost radius that it has been thought prudent to attempt; although a part of the bridge of Neuilly stands, cracked as it is, with a curvature of 250 feet radius; and there is no doubt that a firm structure, well arranged in the beginning, might safely be made much flatter than this, if there were any necessity for it.

An elliptical arch would certainly approach nearer to the form of the curve of equilibrium, which would remain little altered by the change of that of the arch; and the pressure might be more equably and advantageously transmitted through the blocks of such an arch, than in the proposed form of the structure. The duration would probably be proportional to the increased firmness of the fabric, and the greater flatness at the crown might allow a wider space for the passage of the masts of large ships on each side of the middle. There might be some additional trouble and expense in the formation of portions of an elliptical curve; but even this might be in a great measure avoided by employing portions of three circles of different radii, which would scarcely be distinguishable from the ellipsis itself.

Those who have imagined that a circular arch must in general be "stronger than an elliptical arch of the same height and span," have not adverted to the distinction between the apparent curvature of the arch, and the situation of the true curve of equilibrium, which depends on the distribution of the weight of the different parts of the bridge, and by no means on the form of the arch-stones only; this form being totally insufficient to determine the true radius of curvature, which is immediately connected with the lateral thrust, and with the strength of the fabric.

QUESTION 8. *Is it necessary or advisable to have a model made of the proposed bridge, or any part of it, in cast iron. If so, what are the objects to which the experiments should be directed; to the equilibration only, or to the cohesion of the several parts, or to both united, as they will occur in the intended bridge?*

Experiments on the equilibration of the arch would be easy and conclusive; on the cohesion or connexion of the parts, extremely uncertain; the form and proportion of the joints could scarcely be imitated with sufficient accuracy; and since the strength of some of the parts concerned, would vary as the thickness simply, and that of others as the square or cube of the thickness, it would be more difficult to argue from the strength of the model upon that of the bridge, than to calculate the whole from still more elementary experiments. Some such experiments ought, however, to be made, on the force required to crush a block of the substance employed; and the form calculated to afford the proper equilibrium, might be very precisely and elegantly determined, by means of the method first suggested by Dr Hooke, that of substituting for the blocks, resting on each other and on the abutments, as many similar pieces forming a chain, and suspended at the extremities. It would, however, be important to make one alteration in the common mode of performing this experiment, without which it would be of little or no value; the parts corresponding to the blocks of the arch, should be formed of their proper thickness and length, and connected with each other and with the abutments by a short joint or hinge in the middle of each, allowing room for a slight degree of angular motion only; and every other part of the structure should be represented in its proper form and proportion and connexion, that form being previously determined as nearly as possible by calculation; and then, if the curve underwent no material alteration by the suspension, we should be sure that the calculation was sufficiently correct; or, if otherwise, the arrangement of the materials might be altered, until the required curve should be obtained; and the investigation might be facilitated by allowing the joints or hinges, connecting the block, to slide a little along their surfaces, within such limits as would be allowable, without too great a reduction of the powers of resistance of the blocks.

QUESTION 9. *Of what size ought the model to be made, and what relative proportions will experiments, made on the model, bear to the bridge when executed?*

The size is of little importance, and it would be unsafe to calculate the strength of the bridge from any general comparison with that of the model. There is an *Essay* of Euler in the *New Commentaries of the Royal Academy of Petersburgh* (Vol. XX. p. 271.), relating expressly to the mode of judging of the strength of a bridge from a model; but it contains only an elementary calculation, applicable to ropes and simple levers, and by no means comprehending all the circumstances that require to be considered in the structure of an arch.

QUESTION 10. *By what means may ships be best directed in the middle stream, or prevented from driving to the side, and striking the arch; and what would be the consequence of such a stroke?*

For the direction of ships, Professor Robison's suggestion seems the simplest and best, that they might be guided by means of a small anchor, dragged along the bottom of the river. The stroke of a ship might fracture the outer ribs, if they were too weak, but could scarcely affect the whole fabric in any material degree, supposing it to be firmly secured by oblique bars, crossing from one side of the abutment to the other side of the middle; and if still greater firmness were wanted, the braces might cross still more obliquely, and be repeated from space to space.

A ship moving with a velocity of three miles in an hour, or about four feet in a second, would be stopped by a force equal to her weight, when she had advanced three inches with a retarded motion; and the bridge could not very easily withstand, at any one point, a force much greater than such a shock of a large ship, if it were direct, without being dangerously strained. But we must consider that a large ship could never strike the bridge with its full force, and that the mast would be much more easily broken than the bridge. The inertia of the parts of the bridge, and of the heavy materials laid on it, would enable it to resist the stroke of a small mass with great mechanical advantage. Thus the inertia of an anvil, laid on a man's chest, enables him to support a blow on the anvil, which would be fatal without such an interposition, the momentum communicated to the greater weight being always less than twice the momentum of the smaller; and this small increase of momentum being attended by a much greater decrease of energy or impetus, which is expressed by the product of the mass into the square of the velocity, and which is sometimes called the ascending or penetrating force, since the height of ascent or depth of penetration is proportional to it, when the resistance is given. And the same mode of reasoning is applicable to any weight falling on the bridge, or to any other cause of vibration, which is not likely to call forth in such a fabric any violent exertion of the strength of the parts, or of their connections. We must also remember, in appreciating the effect of a stroke of any kind on an arched structure, that something of strength is always lost by too great stiffness; the property of resisting velocity, which has sometimes been called resilience, being generally diminished by any increase of stiffness, if the strength, with respect to pressure, remains the same.

QUESTION 11. *The weight and lateral pressure of the bridge being given, can abutments be made in the proposed situation for London Bridge, to resist that pressure?*

Since this question relates entirely to the local circumstances of the banks of the Thames, the persons, to whom it has been referred, have generally appealed to the stability of St Saviour's Church, in a neighbouring situation, as a proof of the affirmative. And it does not appear that there have been any instances of a failure of piles well driven, in a moderately favourable soil. Professor Robison, indeed, asserts that the firmest piling will yield in time to a pressure continued without interruption; but a consideration of the general nature of friction and lateral adhesion, as well as the experience of ages in a multitude of structures actually erected, will not allow us to adopt the assertion as universally true. When, indeed, the earth is extremely soft, it would be advisable to unite it into one mass for a large extent, perhaps as far as 100 yards in every direction, for such a bridge as that under discussion, by beams radiating from the abutments, resting on short piles, with cross pieces interspersed; since we might combine, in this manner, the effect of a weight of 100,000 tons, which could scarcely ever produce a lateral adhesion of less than 20,000, even if the materials were semifluid; for they would afford this resistance, if they were capable of standing in the form of a bank, rising only one foot in five of horizontal extent, which any thing short of an absolute quicksand or a bog would certainly do in perfect security. The proper direction of the joints of the masonry may be determined for the abutment exactly as for the bridge, the tangent of the inclination being always increased, in proportion to the weights of the successive wedges added to the load; and the ultimate inclination of the curve is that in which the piles ought to be driven; being the direction of the result, composed of the lateral thrust, combined with the joint weight of the half bridge, and the abutment.

QUESTION 12. *The weight and lateral pressure of the bridge being given, can a centre or scaffolding be erected over the river, sufficient to carry the arch, without obstructing the vessels which at present navigate that part?*

There seems to be no great difficulty in the construction of such a centre. When the bridge at Wearmouth was erected, the centre was supported by piles and standards, which suffered ships to pass between them without interruption, and a similar arrangement might be made in the present case with equal facility.

QUESTION 13. *Whether would it be most advisable to make the bridge of cast and wrought iron combined, or of cast iron only? And if of the latter, Whether of the hard white metal, or of the soft grey metal, or of gun metal?*

A bridge well built ought to require no cohesive strength of ties, as Mr Southern has justly observed in his answer to the eighth question; and for repulsive resistance, in the capacity of a shore, cast iron is probably much stronger than wrought. It has also the advantage of being less liable to rust, and of expanding somewhat less by heat than wrought iron. But wherever any transverse strain is unavoidable, wrought iron possesses some advantages, and it is generally most convenient for bolts and other fastenings. The kind of iron called gun metal, is decidedly preferred by the most experienced judges, as combining, in the greatest degree, the properties of hardness and toughness; the white being considered as too brittle, and the grey as too soft. Dr Hutton, however, and Mr Jessop, prefer the grey; and if we allow the strength of the gun metal to be at all comparable to that which Mr Reynolds attributes to it, we must also acknowledge that a much weaker substance would be amply sufficient for every practical purpose, and might deserve to be preferred, if it were found to possess a greater degree of tenacity.

QUESTION 14. *Of what dimensions ought the several members of the iron work to be, to give the bridge sufficient strength?*

See the Answers to Questions 7 and 11.

QUESTION 15. *Can frames of cast iron be made sufficiently correct to compose an arch of the form and dimensions shown in the drawings, so as to take an equal bearing as one frame, the several parts being connected by diagonal braces, and joined by an iron cement, or other substance?*

Professor Robison considers it as indispensable that the frames of cast iron should be ground to fit each other; and a very accurate adjustment of the surface would certainly be necessary for the perfect co-operation of every part of so hard a substance. Probably, indeed, any very small interstices that might be left, would in some measure be filled up by degrees, in consequence of the oxydation of the metal, but scarcely soon enough to assist in bearing the general thrust upon the first completion of the bridge. The plan of mortising the frames together is by no means to be advised, as rendering it very difficult to adapt the surfaces to each other throughout any considerable part of their extent. They might be connected either as in the bridge at Wearmouth, by bars of wrought iron let into the sides, which might be of extremely moderate dimensions; or, as in some still more modern fabrics, by being wedged into the grooves of cross plates, adapted to receive them, which very effectually secure the co-operation of the whole force of the blocks, and which have the advantage of employing cast iron only.

QUESTION 16. *Instead of casting the ribs in frames, of considerable length and breadth, would it be more advisable to cast each member of the ribs in separate pieces of considerable lengths, connecting them together by diagonal braces, both horizontally and vertically?*

No joint can possibly be so strong as a single sound piece of the same metal; and it is highly desirable that the curve of pressure should pass through very substantial frames or blocks, abutting fully on each other, without any reliance on lateral joints; but for the upper parts of the work, single ribs, much lighter than those which form the true arch, would be sufficiently firm.

QUESTION 17. *Can an iron cement be made, which shall become hard and durable, or can liquid iron be poured into the joints?*

Mr Reynolds has observed, that a cement, composed of iron borings and saline substances, will become extremely hard; and it is probable that this property depends on the solidity which is produced by the gradual oxydation of the iron. It would certainly be injurious to the strength of the fabric to interpose this cement between perfectly smooth and solid surfaces; but it might be of advantage to fill up with it any small interstices, unavoidably left between the parts. To pour melted iron into the joints would be utterly impracticable.

QUESTION 18. *Would lead be better to use in the whole or any part of the joints?*

Lead is by far too soft to be of the least use; and a saline cement would be decidedly preferable.

QUESTION 19. *Can any improvement be made in the plan, so as to render it more substantial and durable, and less expensive? And if so, what are these improvements?*

The most necessary alterations appear to be the omission of the upper and flatter ribs; the greater strength and solidity of the lower, made either in the form of blocks or of frames with diagonals; a curvature more nearly approaching to that of the curve of equilibrium, and a greater obliquity of the cross-braces.

It would be necessary to wedge the whole structure very firmly together before the removal of the centres, a precaution which is still more necessary for stone bridges, in which a certain portion of soft mortar must inevitably be employed, in order to enable the stones to bear fully on each other, and which has been very properly adopted in the best modern works. In this manner we may avoid the inconvenience pointed out by Professor Robison, who has remarked, that the compressibility of the materials, hard as they appear, would occasion a reduction of three inches in the length of the bridge, from the effect of the lateral thrust, and a consequent fall at the crown of 15; a result which will not be found materially erroneous, if the calculation be repeated from more correct elements, derived from later experiments and comparisons. For obviating the disadvantageous effects of such a depression, which he seems to have supposed unavoidable, as well as those of a change of temperature, which must in reality occur, though to a less considerable extent, Professor Robison suggested the expedient of a joint in the middle of the bridge, with an intermediate portion, calculated to receive the rounded ends of the opposite ribs, somewhat like an interarticular cartilage; but it is impossible to devise any kind of joint, without limiting the pressure, during the change of form, to a very small portion of the surfaces, which could not bear fully on each other throughout their extent, if any such liberty of motion were allowed, unless all friction between them were prevented; and a similar joint would be required at the abutment, where it would be still more objectionable, as extending to a wider surface.

The arrangement of the joints between the portions of the ribs, in one or more transverse lines, would be a matter of great indifference. Some have recommended to break the joints, as is usual in masonry, in order to tie the parts more firmly together; others to make all the joints continuous, as a safer method, on account of the brittleness of the materials; but if the fabric were well put together, there would be neither any want of firm connexion, nor any danger of breaking from irregular strains, in whatever way the joints might be disposed.

QUESTION 20. *Upon considering the whole cir-*

cumstances of the case, agreeable to the Resolutions of the Committee, as stated at the conclusion of their Third Report, is it your opinion, that an arch of 600 feet in the span, as expressed in the drawings produced by Messrs Telford and Douglas, or the same plan, with any improvement you may be so good as to point out, is practicable and advisable, and capable of being made a durable edifice?

The answers that have been returned to this question are almost universally in the affirmative, though deduced from very discordant and inconsistent views of the subject. The only reasonable doubt relates to the abutments; and with the precautions which have been already mentioned in the answer to the 11th question, there would be no insuperable difficulty in making the abutments sufficiently firm.

QUESTION 21. *Does the estimate, communicated herewith, according to your judgment, greatly exceed or fall short of the probable expense of executing the plan proposed: specifying the general grounds of your opinion?*

The estimate amounts to L. 262,289; and it has generally been considered as below the probable expense. The abutments are set down at L. 20,000; but they would very possibly require five times as much, to be properly executed; while some other parts of the work, by a more judicious distribution of the forces concerned, might safely be made so much lighter, as considerably to lessen the expense of the whole fabric, without any diminution either of its beauty or of its stability.

SECTION VI.—*Modern History of Bridges.*

The whole series of the questions, which we have been considering, are fully as interesting at the present moment, as they were at the time when they were circulated by the Committee of the House of Commons. The practice of building iron bridges has been progressively gaining ground, ever since its first introduction in 1779, by Mr Abiah Darby of Colebrook Dale. Mr Wilson, indeed, who assisted Mr Burdon in the erection of the bridge at Wearmouth, mentions in his answers, an iron bridge which has stood secure for ninety years: but it must have been on a very small scale, and has not been at all generally known. Of most of the later iron bridges we find a concise account in Dr Hutton's elaborate Essay on Bridges, which has been reprinted in the first volume of his valuable collection of Tracts: but there are some still greater edifices of this kind which still remain to be completed.

Mr Darby's construction is not remarkably elegant (Plate XLII. fig. 8.), but it is by no means so objectionable as several late authors have seemed to think it. The span is 100 feet 6 inches: the weight 178½ tons. The curvature of the exterior concentric arches, which assist in supporting the roadway, though it may be somewhat too great for the most favourable exertion of their resistance, leaves them still abundantly strong for the purpose intended; nor is it correct to say that every shore supporting a pressure should be straight; for if its own weight bears any considerable proportion to that which it has to support, the curvature ought to be the same with that of a chain of the same weight, suspending a similar load in an inverted position: and the parts of the bridge in question seem to differ only about as much from such a form in excess of curvature, as a straight line would differ from it in defect. The partial failure, which accidentally occurred, rather bears testimony to the merits than to the demerits of the bridge, as they would be estimated in any other situation: for the lateral thrust, which it is generally desirable to reduce as much as possible, was here actually too small, and the abutments were forced inwards, by the external pressure of the loose materials, forming the high banks, against which the abutments rested.

Mr Paine's iron bridge, exhibited in London, and intended to have been erected in America, was a professed imitation of a catenarian curve: it was a good specimen of that ideal something, which a popular reformer generally has in view: a thing not ill imagined, and which might possibly succeed very well under very different circumstances; but which, when closely examined, proves to be wholly unfit for the immediate purpose to which the inventor intends to apply it.

The bridge at Wearmouth was completed in 1796, in great measure through the exertions of Mr Burdon, both as architect and as principal proprietor of the undertaking. It is remarkable for springing 70 feet above low water mark; and the arch rises 30 feet, leaving a height of 100 feet in the whole for the passage of ships in the middle of the stream: the span is 240. The abutments are founded on a solid rock, but their own internal solidity appears to be somewhat deficient. The weight of iron is 250 tons; 210 of them being of cast iron, and 40 of wrought. (Plate XLII. fig. 9.)

A bridge was finished in the same year at Buildwas, near Colebrook Dale (Plate XLII. fig. 10.), under the direction of Mr Telford; 130 feet in span, weighing 174 tons; and rising only 17 feet in the roadway, but furnished on each side with a stronger arch, of about twice the depth, which extends to the top of the railing, and assists in suspending the part of the road which is below it by means of king-posts, and in supporting the part nearer the abutments by braces and shores. The breadth is only 18 feet; and the construction would not be so easily applicable to a wider bridge, unless the road were divided in the middle by an additional elevated arch with its king-posts, like the celebrated wooden bridge at Schafhausen, which was burnt down by one of the French armies. A third iron bridge was also erected in 1796 on the Parrot at Bridgewater, by the Colebrook Dale Company. It consists of an elliptic arch, of 75 feet span, and 23 feet height, and somewhat resembles the bridge at Wearmouth in the mode of filling the haunches with circular rings: a mode not very advantageous for obtaining the greatest possible resistance from the materials, and consequently throwing a little too much weight on the parts of the arch which support them; although it is probable that no great inconvenience has actually arisen from this cause.

An attempt was also made, about the same time, to throw an iron bridge over the river Tame in Herefordshire; but it fell to pieces as soon as the centre was removed. A similar failure occurred some time afterwards in a bridge of about 180 feet span, which was erected on the Tees at Yarm. In 1802 or 1803, an elegant iron bridge, of 181 feet span, and 16½ rise, was erected at Staines. Its general form resembled that of the bridge at Wearmouth, but the mode of connexion of the parts was somewhat different. In a short time after its completion, it began to sink, and some of the tranverse pieces broke, in consequence of the change of form. Upon examination it was found that one of the abutments had given way: and when this was repaired and made firmer, the other failed. The abutment was pushed outwards horizontally, without any material derangement of its form or direction; a circumstance which could not have happened if its weight had been sufficiently great: but the architect seems to have trusted to the firmness of the iron, and the excellence of the workmanship, and to have neglected the calculation of the lateral thrust, which it is of so much importance to determine.

Mr Rennie has executed several iron bridges with success in Lincolnshire; one at Boston, over the Witham, of which the span is 86 feet, and the rise 5½ only: but the abutments being well constructed, it has stood securely, notwithstanding the fracture of some of the cross pieces of the frames, which had been weakened by the unequal contraction of the metal in cooling. At Bristol, Messrs Jessop erected two iron bridges, of 100 feet span, rising 15; each of them contains 150 tons of grey iron; and the expense of each was about L.4000. The construction appears to be simple and judicious. (Plate XLII. fig. 11.)

Mr Telford has been employed in the construction of several aqueduct bridges on a considerable scale. One of these was cast by Messrs Reynolds, and completed in 1796, near Wellington in Shropshire: it is 180 feet long, and 20 feet above the water of the river, being supported on iron pillars. Another, still larger, was cast by Mr Hazledine, for carrying the Ellesmere canal over the river Dee, at Pontcysylte, in the neighbourhood of Llangollen. It is supported, 126 feet above the surface of the river, by 20 stone pillars, and is 1020 feet in length, and 12 feet wide. (Plate XLII. fig. 12.)

In France, a light iron bridge, for foot passengers only, was thrown across the Seine, opposite to the gate of the Louvre, in 1803. It is supported by stone piers, which are too narrow to withstand the effect of an accident happening to any part of the fabric, and leaving the lateral thrust uncompensated: nor is there any immediate reason to apprehend that any inconvenience should arise from this deficiency of strength; since it is highly improbable than any partial failure should occur, in such a situation, supposing the bridge originally well constructed. (Plate XLIII. fig. 1.)

But all these works have been far exceeded, in extent and importance, by the three new bridges, lately built and now building over the Thames. The Vauxhall Bridge was completed and opened in August 1816: it consists of nine arches of cast iron, each of 78 feet span, and between 11 and 12 feet rise. The breadth of the roadway is 36 feet clear. The architect was Mr Walker. The form of the arches considerably resembles that of Messrs Jessop's bridges at Bristol; but it is somewhat lighter and more elegant, and it has the advantage of a greater solidity in the blocks supporting the principal part of the pressure. (Plate XLIII. fig. 2, 3.)

This advantage characterizes also very strongly the masterly design of Mr Rennie for the structure about to be erected at the bottom of Queen Street, Cheapside, opposite to Guildhall, under the name of the Southwark Bridge. It exhibits an excellent specimen of firmness of mutual abutment in the parts constituting the chief strength of the arch, which has been shown in this essay to be so essential to the security of the work, and which the architect has probably been in great measure induced to adopt from his practical experience of the comparative merits of different arrangements. A plan of the bridge was in February last made public in the Repertory of Arts; a work which amply deserves the encouragement of all those who wish to promote the diffusion of useful information: and the magnitude of the object is such, as to justify our entering into some details of calculation respecting the pressure and strength of the different parts of the fabric, founded on a particular account of their weights and dimensions, which has not yet been made public. (Plate XLIII. fig. 4, 5, 6.)

An act of Parliament for the erection of this bridge was passed in 1811; but it was not begun till 1814; the act having directed that no operations should be commenced, until L. 300,000, out of the required L. 400,000, should be raised by subscription. The subscribers are allowed to receive ten *per cent.* annually on their shares, and the remainder of the receipts is to be laid by, and to accumulate, until it shall become sufficient to pay off to the proprietors the double amount of their subscriptions, and after this time the bridge is to remain open, without any toll. A considerable part of the iron work is already cast, by Messrs Walkers of Rotherham. The middle arch is to be 240 feet in span, the side arches 210 feet each. The abutment is of firm masonry, connected by dowels, to prevent its sliding; and resting on gratings of timber, supported by oblique piles. The piers stand on foundations nine or ten feet below the present bed of the river, in order to provide against any alterations which may hereafter take place in its channel, from the operation of various causes: and they are abundantly secured by a flooring of timber, resting on a great number of piles.

Weight of half of the middle arch of Southwark Bridge.

No.	8 Blocks.		5 Oblique Stays.		Cross frames.		Crosses.		Spandrils.		Total.	
	t.	*cwt.*	*t.*	*cwt.*	*t.*	*cwt.*	*t.*	*cwt.*	*t.*	*cwt.*	*t.*	*cwt.*
1	62	18	2	11	11	0	9	1	26	4	111	17
2	60	19	2	12	10	13	8	15	20	3	103	4
3	54	15	2	13	10	2	8	3	32	16	108	10
4	51	3	2	11	9	17			23	14	87	6
5	50	17	2	13	9	15			32	14	95	19
6	51	2	2	13	9	15			24	15	88	6
half 7	25	12	2	12					20	7	48	12
(Carry forward,)											643	15

	Total. t. cwt.
(Brought forward,)	643 15
Covering-plates - - -	152 0
Cornice and palisades - -	77 5
Roadway and pavement - -	650 0
Whole weight - - - -	1523 0
Springing plate - - - -	13 10
Abutment - - - -	11,000 0

Span 240 feet. Rise 24. Depth of the blocks or plates at the crown 6 feet; at the pier 8 feet.

It is evident from the inspection of this statement of the weights, that their distribution is by no means capable of being accurately expressed by any one formula; but it will be amply sufficient for the determination of the thrust, to employ the approximation founded on the supposition of a parabolic curve (Prop. T.); and if we afterwards wished to find the effect of any local deviation from the assumed law of the weight, we might have recourse to the mode of calculation exemplified in the answer to the fifth Question. But, in fact, that answer may of itself be considered as sufficient to show, that the effect of a variation of a few tons, from the load appropriate to each part, would be wholly unimportant.

We must, therefore, begin by finding the weight of a portion of the arch corresponding to a quarter of the span; and the whole angle, of which the tangent is $\frac{24}{120} = .2$, being $11^0\ 18\frac{1}{2}'$, its sine is .1961; and the angle, of which the sine is .09805, being $5^0\ 37\frac{1}{2}$, we have to compute the weight of $\frac{337.5}{678.5}$, or $\frac{1}{2.01}$, of the angular extent, beginning from the middle of the arch. And this will be $48\frac{12}{20} + 88\frac{6}{20} + 95\frac{12}{20} + (87\frac{6}{20}) \times .7345 = 297$ tons. Now, the weight of the covering-plates, cornice, palisades, roadway, and pavement, are distributed throughout the length, without sensible inequality, making 879 tons; from which the part immediately above the piers might be deducted; but it will be safer to retain the whole weight, especially as something must be allowed for the greater extent of the upper surface of the wedges. We shall, therefore, have, for the interior quarter, $297 + 439.5 = 736.5$ tons, and for the exterior $1523 - 736.5 = 786.5$, the difference being 50 tons; one-sixth of which, added to 736.5, gives us 744.8 for the reduced weight, which is to the lateral thrust as the rise to the half span. But for the rise we must take 23 feet, since the middle of the blocks next to the piers is a foot more remote from the intrados than that of the blocks at the crown. And the true half span, measured from the same point, will be $4 \times \frac{120}{312}$ greater than that of the intrados, amounting to 121.6. We have, therefore, $23 : 121.6 = 745.8 : 3942$ tons, for m the lateral thrust. And for $\frac{1}{2}ax$, $736.5 - \frac{50}{6} = 728.2$; whence, $\frac{1}{2}x$ being 60.8, $a = 11.98$, and $r = \frac{m}{a} = 329$ feet, the radius of curvature of the curve of equilibrium at the vertex, while that of the middle of the blocks is 334. In order to determine the ordinate y, we have $my = \frac{1}{2}ax^2 + \frac{1}{12}bx^4$; but $\frac{1}{2}ax$ for the whole arch is 728.2, and $\frac{1}{4}bx^3 = 50$; consequently $my = 728.2x + \frac{50}{3}x$, the first portion varying as x^2, and the second as x^4; and the sum y being $23 = 22.49 + .51$, the ordinate at $\frac{1}{4}x$ or 30.4 feet is $\frac{1}{16} \times 22.49 + \frac{1}{256} \times .51 = 1.41$; and, in a similar manner, any other ordinate may be calculated, so that we have,

x.	y.	Middle of the Blocks.
30.2	1.41	1.40
60.8	5.65	5.67
91.0	13.02	12.89
121.6	23.00	23.00

Hence it appears that the curve of equilibrium nowhere deviates more than about two inches from the middle of the blocks, which is less than one fortieth of the whole depth.

The half weight of the smaller arches is probably about 1300 tons, and their lateral thrust 3500; and, since the abutment weighs 11,000 tons, the foundation ought to have an obliquity of $\frac{3500}{12300}$, or more than 1 in 4, if it were intended to stand on the piles without friction; but in reality it rises only 66 inches in 624, or nearly 1 in 9; so that there is an angular difference of 1 in 7 between the direction of the piles and that of the thrust, which is probably a deviation of no practical importance.

It remains to be inquired how far the series of masses of solid iron, constituting the most essential part of the arch, is well calculated to withstand the utmost changes of temperature that can possibly occur to it in the severest seasons (Prop. K.) For this purpose, we may take the mean depth $a = 7$ feet, h being 23; then $1 + \frac{4h}{a} = \frac{99}{7} = 14.14$, and $1 + \frac{16hh}{15aa} = \frac{9199}{735} = 12.52$: consequently the greatest actual compression or extension of such a structure is to the mean change which takes place in the direction of the chord, as 14.14 to 12.52, or as 1.129 to 1; and if, in a long and severe frost, the temperature varied from 52° to 20°, since the general dimensions would contract about $\frac{1}{5000}$, the extreme parts of the blocks near the abutments would vary $\frac{1.129}{5000}$ of their length; and the modulus M being about 10,000,000 feet, this change would produce a resistance equivalent to the weight of a column of the same substance 2258 feet high: that is, to about three tons for each square inch, diminishing gradually towards the middle of the blocks, and converted on the other side into an opposite resistance: so that this force would be added to the general pressure below in case of contraction, and above in case of extension. Now, the lateral thrust is derived from a pressure equivalent to a column about 329 feet high, of materials weighing 1523 tons, while the blocks themselves weigh 357; that is, to a column equal in section to the blocks, and 1400 feet high: it will, therefore, amount to about two tons on each square inch: consequently such a change of temperature, as has been supposed, will cause the extreme parts of the abutments to bear a pressure of five tons, where, in the ordinary circumstances, they have only to support two.

The ingenious architect proposes to diminish this contingent inconvenience, by causing the blocks to bear somewhat more strongly on the abutments at the middle than at the sides, so as to allow some little latitude of elevation and depression, in the nature of a joint: and, no doubt, this expedient will prevent the great inequality of pressure which might otherwise arise from the alternations of heat and cold. But it cannot be denied that there must be some waste of strength in such an arrangement, the extreme parts of the abutments, and of the blocks near them, contributing very little to the general resistance; and when we consider the very accurate adjustment of the equilibrium throughout the whole structure, we shall be convinced that there is no necessity for any thing like so great a depth of the solid blocks, especially near the abutments; and that the security would be amply sufficient if, with the same weight of metal, they were made wider in a transverse direction, preserving only the form of the exterior ones on each side, if it were thought more agreeable to the eye. In carpentry, where there is often a transverse strain, and where stiffness is frequently required, we generally gain immensely by throwing much of the substance of our beams into the depth; but in a bridge perfectly well balanced, there is no advantage whatever from depth of the blocks: we only want enough to secure us against accidental errors of construction, and against partial loads from extraneous weights; and it is not probable that either of these causes, in such a bridge, would ever bring the curve of equilibrium six inches, or even three, from its natural situation near the middle of the blocks.

We cannot conclude our inquiries into this subject with a more striking example, than by applying the principles of the theory to the magnificent edifice which is now nearly finished, by the same judicious and experienced architect, and which is destined to bear the triumphant appellation of Waterloo Bridge; a work not less pre-eminent among the bridges of all ages and countries, than the event which it will commemorate is unrivalled in the annals of ancient or modern history. It consists of nine elliptical arches, each of 120 feet span, and 35 feet rise. The piers are 20 feet thick, the road 28 feet wide, besides a foot pavement of seven feet on each side. The arches and piers are built of large blocks of granite, with short counterarches over each pier. The haunches are filled up, as is usual in the most modern bridges, by spandrils, or longitudinal walls of brick, covered with flat stones, and extending over about half the span of the arch; the remainder being merely covered with earth or gravel, which is also continued over the stones covering the spandrils. The hollow spaces between the walls are carefully closed above, and provided with outlets below, in order to secure them from becoming receptacles of water, which would be injurious to the durability of the structure. The mean specific gravity of the materials is such, that a cubic yard of the granite weighs exactly two tons, of the brick work one ton, and of the earth a ton and an eighth. Hence, the weight of the whole may be obtained from the annexed statement. (Plate XLIV. fig. 1, 2, 3.)

Contents of the materials in half an arch of Waterloo Bridge, from the middle of the pier to the crown, beginning from the springing of the arch.

		Cubic Feet.
Half of the arch stones,		25311
Half of the inverted arch,		2555
Square spandril between them,		1994
Outside spandril walls,		4374
Spandrils of brick,	4976	(= 2489)
Kirbels of the brick spandrils,		1271
Flat stone covers,		969
Earth,	10260	(= 5771)
Foot pavement,		620
Frises, E. and W.		1586
Cornice, E. and W.		1120
Plinth of balustrade,		510
Solid in parapet,		416
Balusters 72, 151 cwt.		102
Coping, E. and W.		142

From this statement, and from a consideration of the arrangement of the materials, exhibited in the plate, we may infer that the half arch, terminated where the middle line of the arch-stones enters the pier, is equivalent in weight to about 34,000 cubic feet of granite; its inner half containing in round numbers 13,000, and its outer 21,000, whence we have 14,333 for the reduced weight of the quarter arch (Prop. T.). The extreme ordinate will be about 21 feet; the middle of the blocks being somewhat more than 16 feet above the springing of the arch, and the key-stone being four feet six inches deep; consequently the horizontal thrust will be expressed by $14,333 \times \frac{60}{21} = 40,952$ cubic feet, weighing 3033 tons. But $\frac{1}{2} ax$ being 11667, and $\frac{1}{2} x = 30$, $a = 389$, and $\frac{m}{a} = r = \frac{40952}{388} = 105$ feet; while the radius of curvature of the ellipsis at the crown is $\frac{60 \times 60}{35} = 103$ feet. It is obvious, therefore, that the curve of equilibrium will pass everywhere extremely near to the middle of the blocks, and there can be no apprehension of any deficiency in the equilibrium. It is true that, as it approaches to the piers, it acquires an obliquity of a few degrees to the joints; but the disposition to slide would be abundantly obviated by the friction alone, even if the joints were not secured by other precautions.

In building the arches, the stones were rammed together with very considerable force, so that, upon the removal of the centres, none of the arches sunk more than an inch and a half. In short, the accuracy of the whole execution seems to have vied with the beauty of the design, and with the skill of the arrangement, to render the Bridge of Waterloo a monument, of which the metropolis of the British Empire will have abundant reason to be proud, for a long series of successive ages.

EXPLANATION OF THE PLATES.

Plate XLII. fig. 1. If AB represent the distance of any two particles of matter, and BC, DE, FG the repulsive forces at the distances AB, AD, AF respectively, and BC, DH, FI, the corresponding cohesive forces, then GI must be ultimately to EH as FB to BD. (Sect. I. Prop. A.)

Fig. 2. The block will support twice as great a pressure applied at A as at B. (Prop. B.)

Fig. 3. It is obvious that $ABC - ADE = ABC - CFG$, HI being $= HK$, and $HG = HA$; and the difference ABFHA is always equal to $DB \times KH$. (Prop. C.)

Fig. 4. It is evident that AB is to CD as AE to CE, or as $z + \frac{1}{2}a$ to z. (Prop. E.) It is also obvious that as z or CE is to CD, so is EF to FG. (Prop. F.)

Fig. 5. Supposing the arch AB to be so loaded in the neighbourhood of C as to require the curve of equilibrium to assume the form ADCEB, the joints in the neighbourhood of D will be incapable of resisting the pressure in the direction of the curve CD, and must tend to turn on their internal terminations as centres, and to open externally. (Prop. Y.)

Fig. 6. A, B, C, Different steps in the fall of a weak arch. (Prop. Y.)

Fig. 7. Elevation and plan of Mess. Telford and Douglas's proposed iron-bridge over the Thames. (Sect. V.)

Fig. 8. Elevation of Mr Darby's iron Bridge at Colebrook Dale. (Sect. VI.)

Fig. 9. Elevation of Mr Burdon's Bridge at Wearmouth. (Sect. VI.)

Fig. 10. Elevation of Mr Telford's Bridge at Buildwas. (Sect. VI.)

Fig. 11. Elevation of Messrs Jessop's Bridges at Bristol. (Sect. VI.)

Fig. 12. Elevation of Mr Telford's Aqueduct Bridge at Pontcysylte. (Sect. VI.)

Plate XLIII. Fig. 1. Elevation of the Bridge of the Louvre at Paris. (Sect. VI.)

Fig. 2. Elevation of Vauxhall Bridge. (Sect. VI.)

Fig. 3. Middle arch of Vauxhall Bridge. (Sect. VI.)

Fig. 4. Middle arch of Southwark Bridge. (Sect. VI.)

Fig. 5. Elevation of Southwark Bridge. (Sect. VI.)

Fig. 6. Plan of Southwark Bridge. (Sect. VI.)

Fig. 7. Elevation of London Bridge in its present state. (Sect. IV.)

Fig. 8. Plan of London Bridge, with its sterlings. (Sect. IV.)

Fig. 9. London Bridge, as proposed by Mr Dance to be altered.

Plate XLIV. Fig. 1. Elevation of Waterloo Bridge (Sect. VI.)

Fig. 2. Plan of Waterloo Bridge. (Sect. VI.)

Fig. 3. Section of an arch of Waterloo Bridge, showing the foundations of the piers, and the spandril walls of brick; together with the centre supporting it. The dotted line represents the direction of the curve of equilibrium. (Sect. VI.) (O. R.)

BRISSON (MATHURIN JAMES), a zoologist and natural philosopher, born at Fontenay le Comte, 3d April 1723, the son of Mathurin Brisson and Louisa Gabrielle Jourdain.

He was originally intended for the church, but he had acquired at an early age a taste for natural history, which was particularly encouraged by the advantage that he enjoyed of passing his holidays with the justly celebrated Réaumur, who had an estate near Fontenay. At the age of twenty-four, he had made great progress in his theological studies, and had fully qualified himself for the rank of a subdeacon; but his courage failed him at the time appointed for taking orders, and he then determined to confine himself to the study of physical sciences. Réaumur had the direction of the Chemical Laboratory of the Academy of Sciences, and had given up the salary attached to it to several young men in succession, whom he appointed as his assistants, and of whom Pitot and Nollet became afterwards the most distinguished. He now chose Brisson for the situation, which served him, as it had done his predecessors, rather as a step in his advancement with respect to general science, than in enabling him to pursue any objects more immediately chemical; and he followed his passion in attaching himself, almost exclusively, to natural history. The collection of Réaumur furnished him with ample materials for his studies, and with the principal subjects described in his works on the *Animal Kingdom*. The first of these was published in 1756, containing quadrupeds and cetaceous animals. It consists of simple descriptions of the different species, together with synonyms in various languages, more in the nature of a prodromus than of a complete history. His *Ornithologie* appeared in 1760, forming six volumes, and containing a number of well-executed plates. But upon Réaumur's death, the collection having been added to the Royal Cabinet, Messrs Buffon and Daubenton, the Directors of that Cabinet, not affording him all

BRIDGE

PLATE XLII.

Fig. 1.

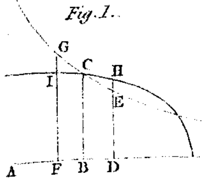


Fig. 2.

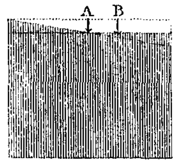


Fig. 3.

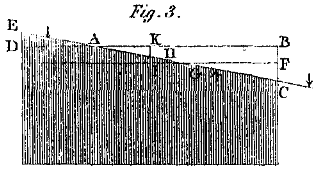


Fig. 5.

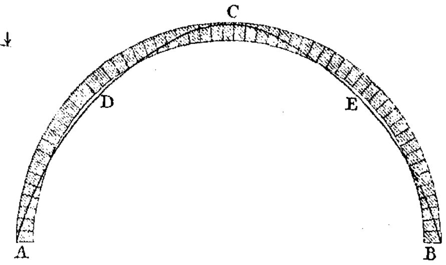


Fig. 4.

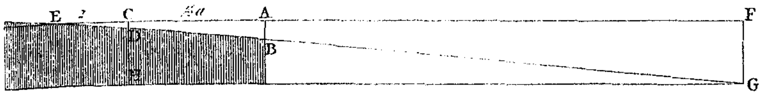
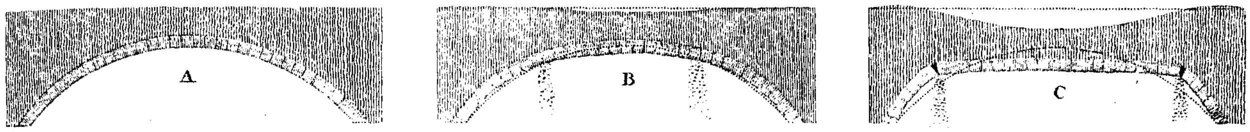
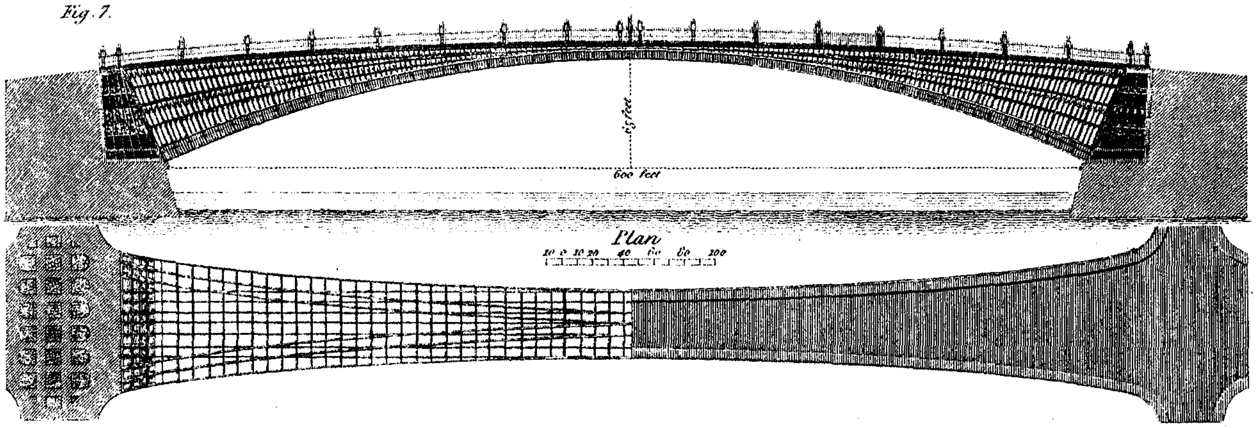


Fig. 6.



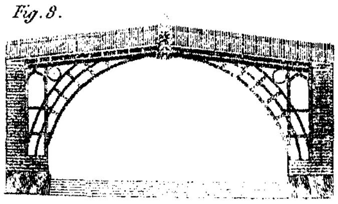
BRIDGE PROPOSED BY MESS: TELFORD AND DOUGLASS.

Fig. 7.



COLEBROOK DALE

Fig. 8.



WEARMOUTH

Fig. 9.

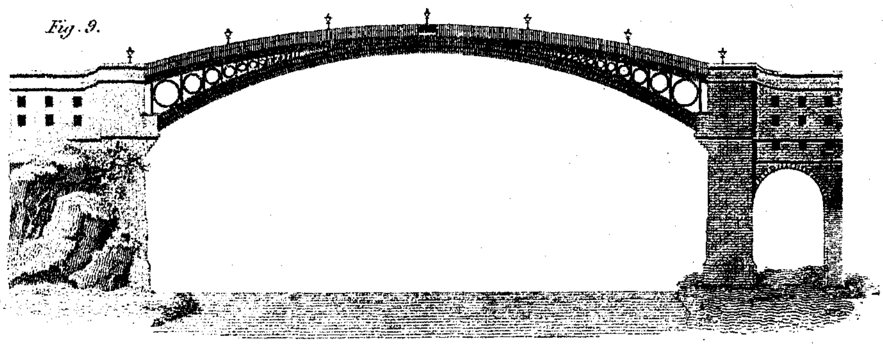


Fig. 10. *BUILDWAS*

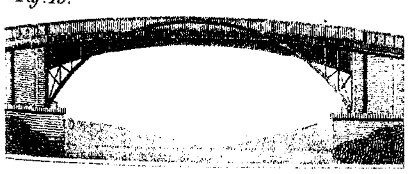


Fig. 11. *BRISTOL*

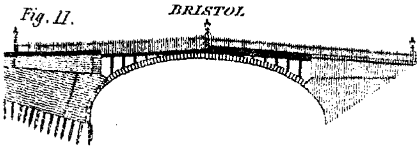
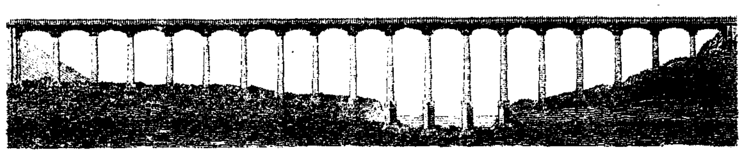


Fig. 12. *PONTCYSYLTE*



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BRIDGE. PLATE XLIII.

Fig. 1.

BRIDGE OF THE LOUVRE

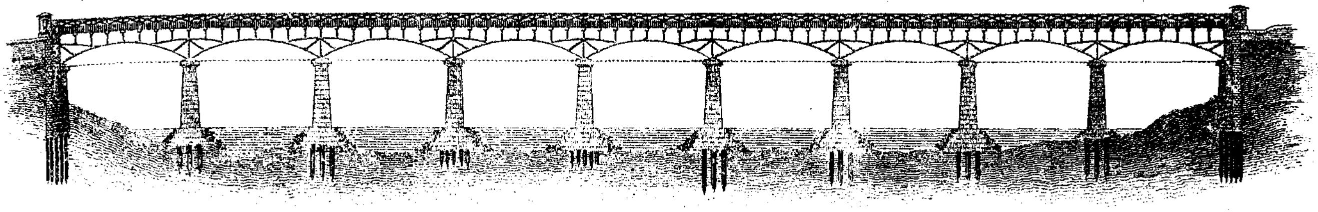


Fig. 2.

VAUXHALL

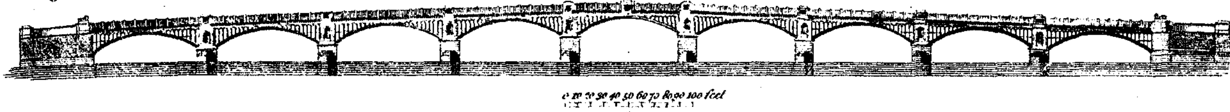


Fig. 3.

MIDDLE ARCH

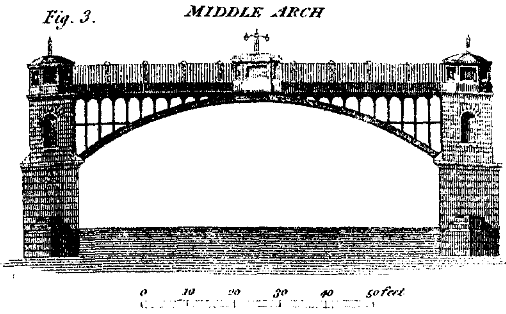


Fig. 4.

SOUTHWARK MIDDLE ARCH

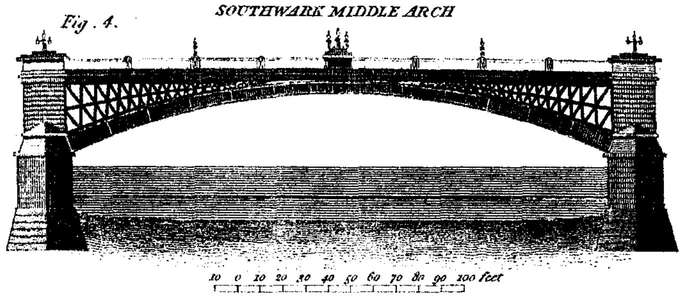


Fig. 5.

SOUTHWARK

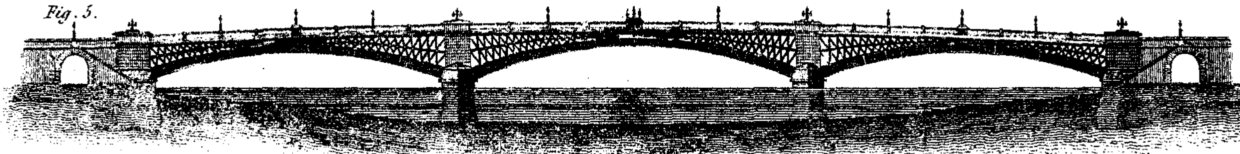


Fig. 6.

Plan

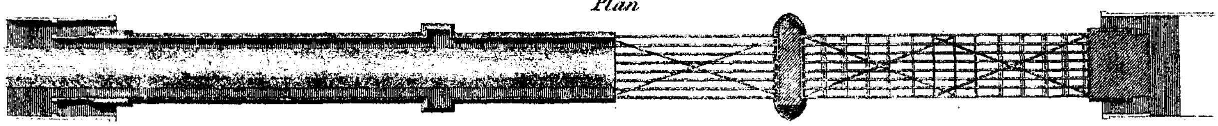


Fig. 7.

LONDON BRIDGE

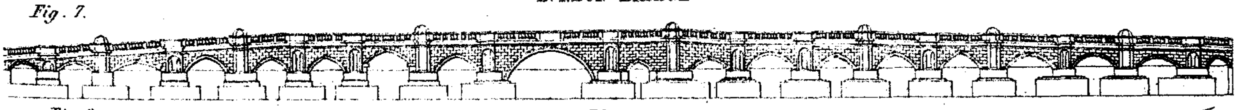


Fig. 8.

Plan

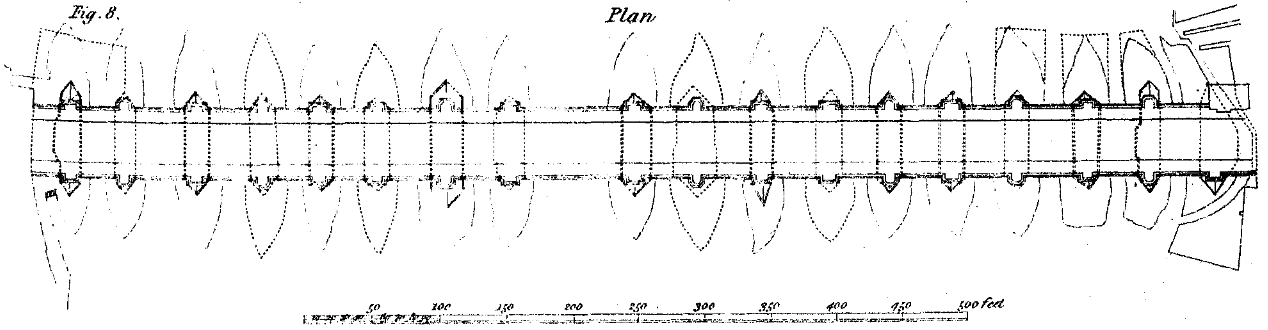
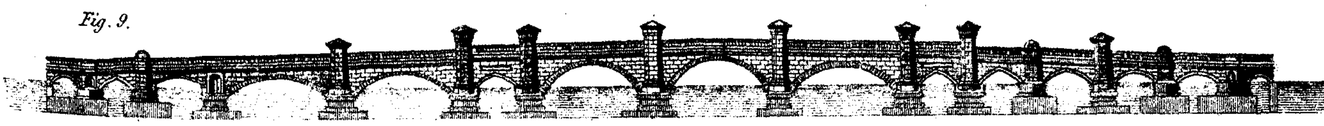


Fig. 9.

PROPOSED IMPROVEMENTS



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WATERLOO BRIDGE.

Fig. 1.

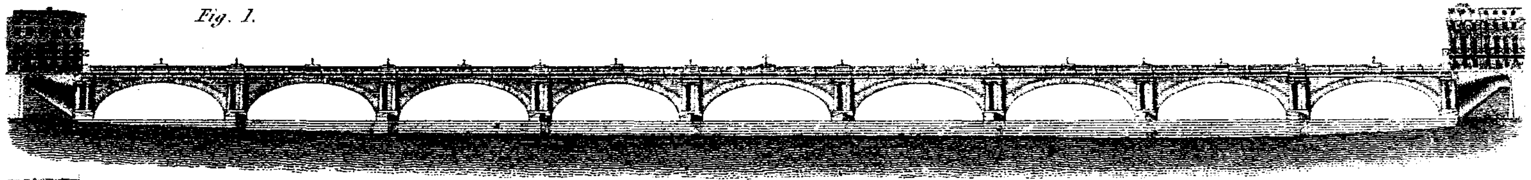


Fig. 2.

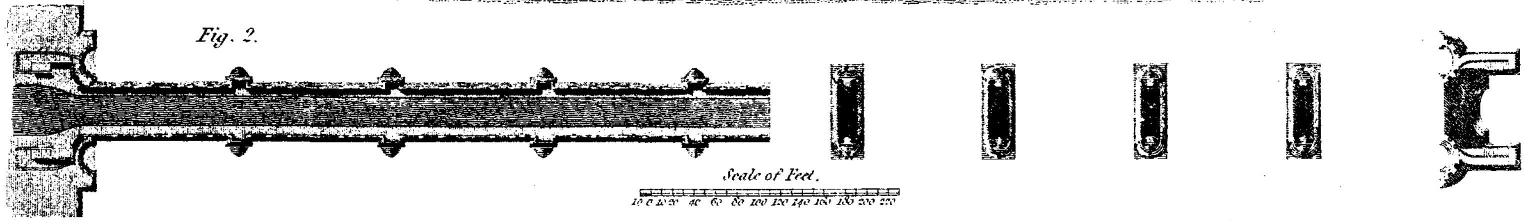
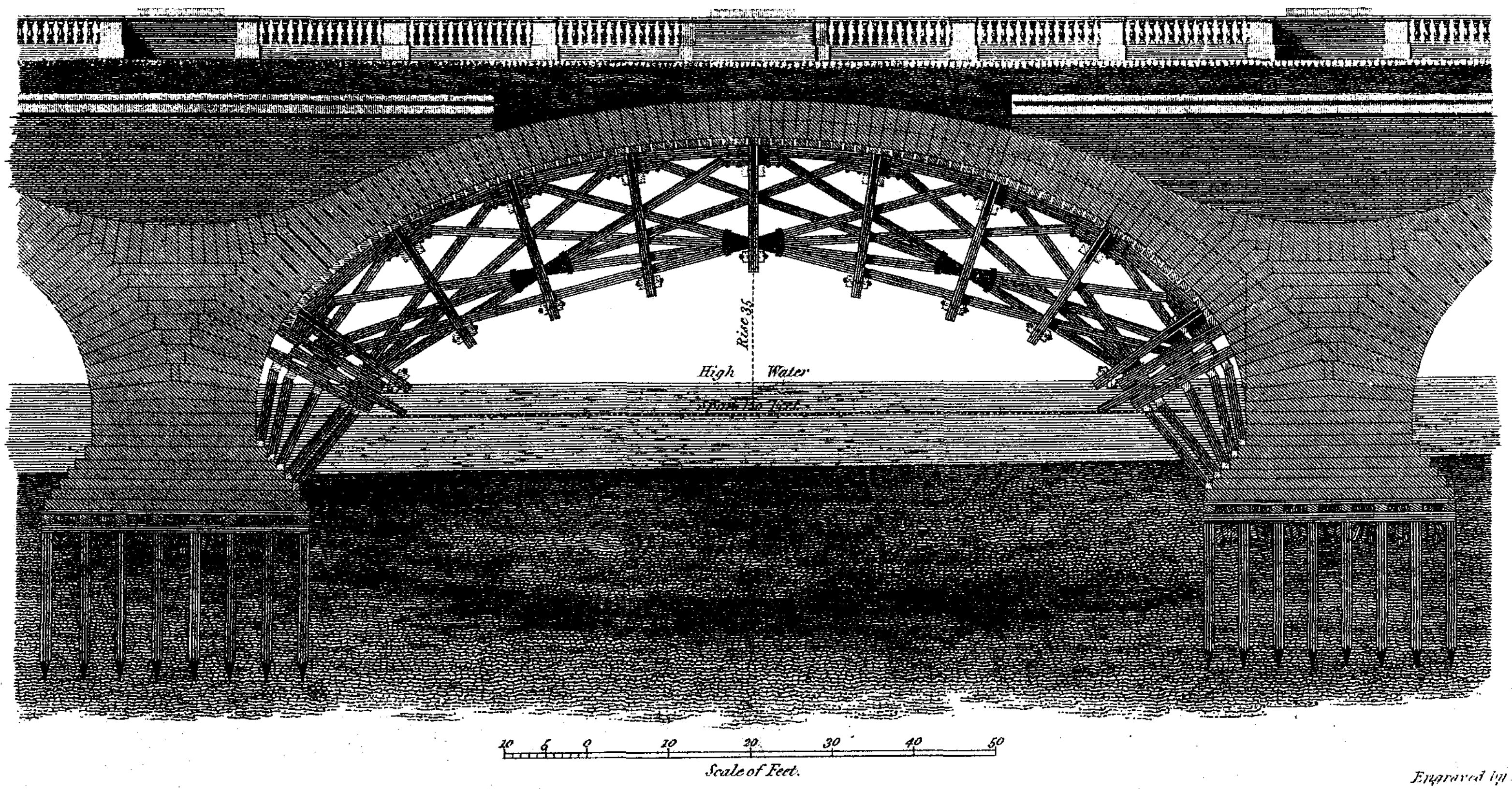


Fig. 3. SECTION OF AN ARCH, WITH THE CENTRE.



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